

# Composite Identities

Mr Murphy

Name Solutions

$$\sin A = -\frac{24}{25}$$

A.M.D.G.

$$\cos A = -\frac{7}{25} \text{ in Quadrant III, } \sin P = -\frac{15}{17} \text{ in Quadrant IV, } \tan B = -\sqrt{3} \text{ in Quadrant II}$$

$$\cos P = \frac{8}{17} \leftarrow$$

$$\frac{y}{x} = \frac{\sqrt{3}}{-1}$$

Use the above information to answer problems 1 and 2

1) Find the value of  $\cos(A+P) = \cos A \cos P - \sin A \sin P$

$$r^2 = (\sqrt{3})^2 + (-1)^2$$

$$r^2 = 4 \quad r = 2$$

$$\sin B = \frac{\sqrt{3}}{2}$$

$$\cos B = -\frac{1}{2}$$

$$= \left(-\frac{7}{25}\right)\left(\frac{8}{17}\right) - \left(-\frac{24}{25}\right)\left(-\frac{15}{17}\right)$$

$$= -\frac{56}{425} - \frac{360}{425} = -\frac{416}{425}$$

2) Find the value of  $\sin(P-A) = \sin P \cos A - \cos P \sin A$

$$= \left(-\frac{15}{17}\right)\left(-\frac{7}{25}\right) - \left(\frac{8}{17}\right)\left(-\frac{24}{25}\right)$$

$$= \frac{105}{425} + \frac{192}{425} = \frac{297}{425}$$

3) Find the value of  $\sin(A-P) = \sin A \cos P - \cos A \sin P$

$$= \left(\frac{8}{17}\right)\left(-\frac{24}{25}\right) - \left(-\frac{15}{17}\right)\left(-\frac{7}{25}\right)$$

$$= -\frac{192}{425} - \frac{105}{425} = -\frac{297}{425}$$

4) Find the value of  $\cos(A-B)$

$$\cos A \cos B + \sin A \sin B$$

$$\tan B = -\frac{\sqrt{3}}{1} \text{ in QII} \quad (\sqrt{3})^2 + 1^2 = r^2$$

$$\sin B = \frac{\sqrt{3}}{2} \quad \cos B = -\frac{1}{2} \quad r = 2$$

$$\left(-\frac{7}{25}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{24}{25}\right)$$

$$\frac{7}{50} - \frac{24\sqrt{3}}{50} = \frac{7 - 24\sqrt{3}}{50}$$

5) Find the two possible solutions for  $0 \leq x \leq 360^\circ$

a)  $\sin 30 \cos 4x - \cos 30 \sin 4x = \frac{\sqrt{2}}{2}$

$\rightarrow \sin(30 - 4x) = \frac{\sqrt{2}}{2}$

$30 - 4x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) \quad \text{Q I, II}$

$30 - 4x = \begin{cases} 45^\circ \pm 360^\circ n \\ 135^\circ \pm 360^\circ n \end{cases}$

$-4x = \begin{cases} 15^\circ \pm 360^\circ n \\ 105^\circ \pm 360^\circ n \end{cases}$

$x = \begin{cases} -\frac{15^\circ}{4} \pm 90^\circ n \\ -\frac{105^\circ}{4} \pm 90^\circ n \end{cases}$

remember that we are also dividing  $360^\circ$  by  $-4$  as well

$x = \begin{cases} -3.75^\circ \pm 90^\circ n \\ -26.25^\circ \pm 90^\circ n \end{cases}$

remember that  $0^\circ \leq x \leq 360^\circ$  so

$x = \begin{cases} -3.75, 86.25, 176.25, 266.25, 356.25 \\ -26.25, 63.75, 153.75, 243.75, 333.75 \end{cases}$

These two answers are  $\leq 0$  so they don't matter here

b)  $\cos 5\theta \cos 15 + \sin 5\theta \sin 15 = -\frac{1}{2}$

note that this is the Greek letter theta not a zero  
 $\cos(5\theta - 15) = -\frac{1}{2}$

$5\theta - 15 = 120^\circ \pm 360^\circ n$

$5\theta = 135^\circ \pm 360^\circ n$

$\theta = 27^\circ \pm 72^\circ n$

$= 25^\circ, 97^\circ, 169^\circ, 241^\circ, 313^\circ$

$5\theta - 15 = 240^\circ \pm 360^\circ n$

$5\theta = 255^\circ \pm 360^\circ n$

$\theta = 51^\circ \pm 72^\circ n$

$= 51^\circ, 123^\circ, 195^\circ, 267^\circ, 339^\circ$

c)  $\sin 2\theta \cos 10 + \cos 2\theta \sin 10 = -\frac{\sqrt{3}}{2}$

$\sin(2\theta + 10) = -\frac{\sqrt{3}}{2}$

$2\theta + 10 = \begin{cases} -60^\circ \pm 360^\circ n \text{ (c.c.)} \\ 240^\circ \pm 360^\circ n \text{ (180-c.c.)} \end{cases}$

$2\theta = \begin{cases} -70^\circ \pm 360^\circ n \\ 230^\circ \pm 360^\circ n \end{cases}$

$\theta = \begin{cases} -35^\circ \pm 180^\circ n \\ 115^\circ \pm 180^\circ n \end{cases}$

$\theta = \begin{cases} -35^\circ, 145^\circ, 425^\circ \\ 115^\circ, 295^\circ \end{cases}$