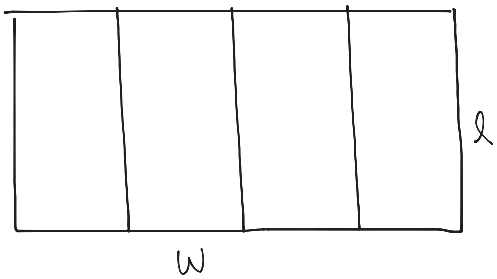
**Optimization** Name________________________

1) Ryan decides to become a farmer and but needs to buy all of his farming supplies on a budget. He only has 750 feet of fencing with which to work and wants to enclose a rectangular area dividing it into a row of four equal subdivisions. Find the maximum total area possible with this much fencing.




$A = w \cdot l \qquad 750 = 2w + 5l$

$l = \frac{750 - 2w}{5}$

$A = w\left(\frac{750-2w}{5}\right)$

$A = 150w - \frac{2}{5}w^2 \rightarrow$ $\boxed{\begin{array}{l} A = 150(\overbrace{187.5}^{w}) - \frac{2}{5}(\overbrace{187.5}^{w})^2 \\ A = \textbf{14062.5 square ft} \end{array}}$

$A' = 150 - \frac{4}{5}w = 0$

$150 = \frac{4}{5}w$

$w = \frac{\cancelto{75}{150} \cdot 5}{\cancelto{2}{4}} = \frac{375}{2} = 187.5 \text{ ft}$

2) Gabby, Blaire, and Ana tell Ryan that he's doing it all wrong and needs to decide on an area first then find the minimum amount of fencing needed to enclose that area. They decide to enclose 15,000 square feet with the same 4 subdivisions. Find the minimum amount of fencing that they will need.

$15{,}000 = w \cdot l$

$\frac{15000}{l} = w$

$F = 2w + 5l$

$F = 2\frac{15000}{l} + 5l$

$F = \frac{30000}{l} + 5l$

$F = 30000l^{-1} + 5l$

$F' = -30000l^{-2} + 5 = 5 - \frac{30000}{l^2} = 0$

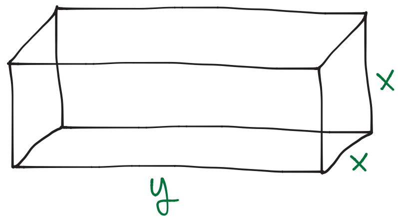
$5 = \frac{30000}{l^2}$

$l^2 = \frac{30000}{5} = 6000$

$l = \sqrt{6000} \approx 77.460 \text{ ft}$

$F_{min} = 774.6 \text{ ft}$

3) Pilar and Hanley are trying to use the least amount of cardboard to make a closed box with square ends and a volume of 8 m$^3$. How much cardboard will they need?




Need to minimize the surface area of the cardboard

$V = 8 = x^2 y$

$\frac{8}{x^2} = y$

$A = 2x^2 + 4xy$

$A = 2x^2 + 4x\left(\frac{8}{x^2}\right)$

$A = 2x^2 + \frac{32}{x} = 2x^2 + 32x^{-1}$

$A' = 4x - 32x^{-2} = 4x - \frac{32}{x^2} = 0$

$4x = \frac{32}{x^2}$

$x^3 = 8 \Rightarrow x = 2$

$A = 2(2)^2 + \frac{32}{2}$

$= 8 + 16 = 24 \text{ ft}^2$

4) Kareen and Madeleine think that they are doing it all wrong and need to use 25 m$^2$ of cardboard and maximize the volume. Will they produce a box with greater volume?

$A = 2x^2 + 4xy$

$V = x^2 y$

$25 = 2x^2 + 4xy$

$\frac{25 - 2x^2}{4x} = y$

$V = x^2\left(\frac{25 - 2x^2}{4x}\right)$

$V = \frac{25x^2 - 2x^4}{4x} = \frac{25x^2}{4x} - \frac{2x^4}{4x}$

$V = \frac{25}{4}x - \frac{1}{2}x^3$

$V' = \frac{25}{4} - \frac{3}{2}x^2 = 0$

$\frac{3}{2}x^2 = \frac{25}{4}$

$x^2 = \frac{25}{\cancel{4}_{2}} \cdot \frac{\cancel{2}}{3}$

$x^2 = \frac{25}{6}$

$x = \pm \frac{5}{\sqrt{6}}$

$V = \frac{25}{4}\left(\frac{5}{\sqrt{6}}\right) - \frac{1}{2}\left(\frac{5}{\sqrt{6}}\right)^3$

$V \approx 8.505 \text{ m}^3$

Yes they will