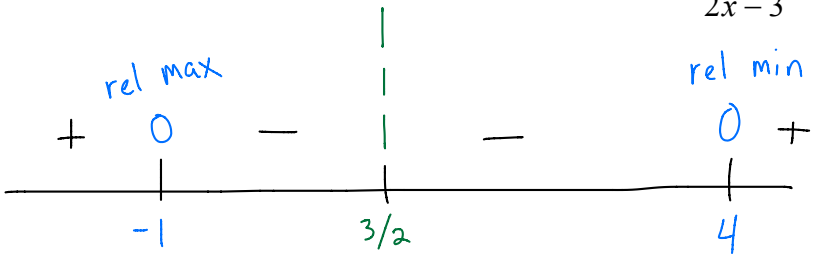
The Quotient Rule (Part 2) **Name** Solutions

Find the extreme values of the rational function as well as the equation of the tangent line at a given point when asked.

1) a) Find the extreme values for the graph of $y = \dfrac{x^2+4}{2x-3}$



$$y' = \frac{(2x-3)(2x) - (x^2+4)(2)}{(2x-3)^2}$$

$$= \frac{4x^2-6x-2x^2-8}{(2x-3)^2} = \frac{2x^2-6x-8}{(2x-3)^2} = 0 \text{ or undef}$$

$$= \frac{2(x^2-3x-4)}{(2x-3)^2} = 0 \text{ or undefined}$$

$x^2-3x-4 = (x-4)(x+1) = 0 \quad x = -1, 4$

$2x-3 = 0 \quad 2x = 3 \quad x = \frac{3}{2} \Rightarrow VA$

Extreme Values

rel max at $(-1,-1)$

rel min at $(4,4)$

b) Find the equation of the tangent line at $x = 4$

$m_T \Rightarrow$ plug $x=4$ into $y' \Rightarrow m_T = \dfrac{2(4)^2-6(4)-8}{(2(4)-3)^2} = \dfrac{2(16)-24-8}{25} = \dfrac{0}{25} = 0$

point $\Rightarrow$ plug $x=4$ into $y = \dfrac{4^2+4}{2(4)-3} = \dfrac{20}{5} = 4$ $\quad y = 4$

2) a) Find the extreme values for the graph of $y = \frac{2x-1}{x^2}$

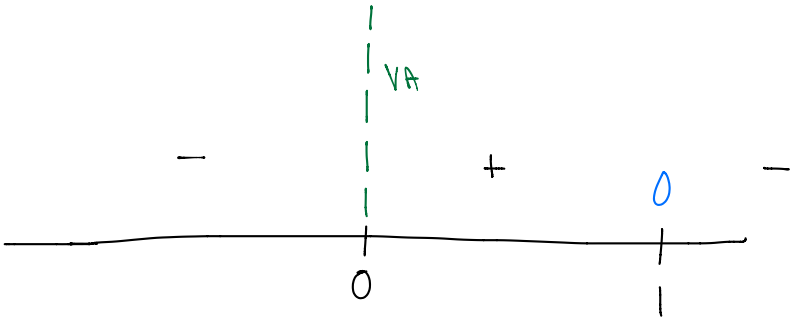
$$y' = \frac{x^2(2) - (2x-1)2x}{x^4}$$

$$= \frac{2x^2 - 4x^2 + 2x}{x^4} = \frac{-2x^2 + 2x}{x^4} = \frac{-2x(x-1)}{x^4}$$

$$= \frac{-2(x-1)}{x^3} = 0 \text{ or undefined}$$

$-2(x-1) = 0 \quad x = 1$

$x^3 = 0 \quad x = 0 \Rightarrow$ VA



$x - 1 = 0 \quad x = 1$

extreme points

(1, 1) relative max

b) Find the equation of the tangent line at $x = 1$

$m_T \Rightarrow$ plug $x = 1$ into $y' \Rightarrow m_T = \frac{-2(1^2-1)}{(1^4} = \frac{0}{1} = 0$

point $\Rightarrow$ plug $x=1$ into $y \Rightarrow y = \frac{2(1)-1}{1^2} = 1$

$y = 1$

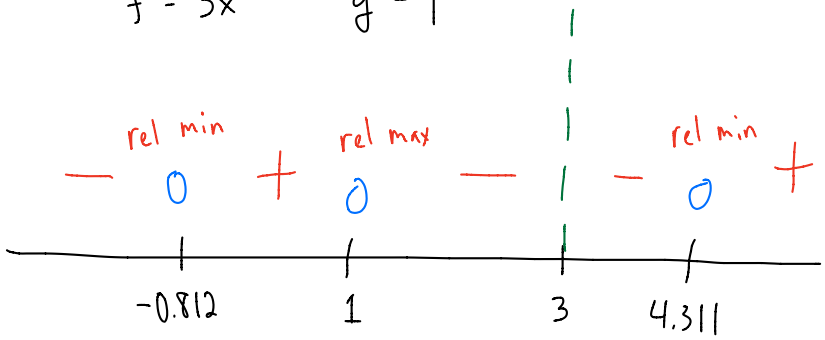
3) Find the extreme values for the graph of $y = \frac{x^3 - 7}{x - 3}$

$f = x^3 - 7 \qquad g = x - 3$

$f' = 3x^2 \qquad g' = 1$

$$y' = \frac{3x^2(x-3) - 1(x^3-7)}{(x-3)^2} = \frac{3x^3 - 9x^2 - x^3 + 7}{(x-3)^2}$$

$$= \frac{2x^3 - 9x^2 + 7}{(x-3)^2} = 0 \text{ or undef} \downarrow x = 3$$



$2x^3 - 9x^2 + 7 = 0 = (x-1)(2x^2 - 7x - 7)$

1 | 2 | −9 | 0 | 7
  | | 2 | −7 | −7
  | 2 | −7 | −7 | 0

$$x = \frac{7 \pm \sqrt{49 + 56}}{4}$$

$$= \frac{7 \pm \sqrt{105}}{4} \approx 4.311, -0.812$$

$x = -0.812, 1, 4.311$

Extreme Values: (−0.812, 1.977) rel min (1, 3) rel max

(4.312, 55.773)