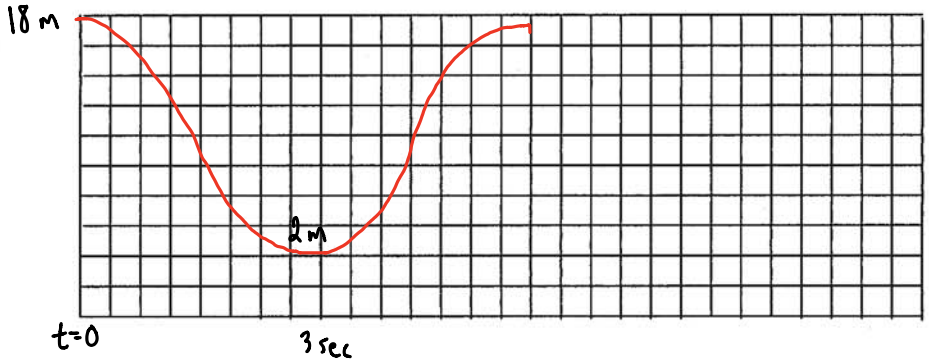
Name: Solutions

Sinusoidal Word Problems

1. In Canada's wonderland there is a roller coaster that is a continuous series of identical hills that are 18m high from the ground. The platform to get on the ride is on top of the first hill. It takes 3 seconds for the coaster to reach the bottom of the hill 2m off the ground

   a) Sketch a graph below which expresses the path of the roller coaster.



   a) What is the sinusoidal equation (sine and cosine) that best reflects this roller coaster's motion?

Max - 18, min - 2 $\Rightarrow a = 16/2 = 8$

period = 6 sec, $b = \frac{2\pi}{6} = \frac{\pi}{3}$

$y = 10 + 8\cos\frac{\pi}{3}t$

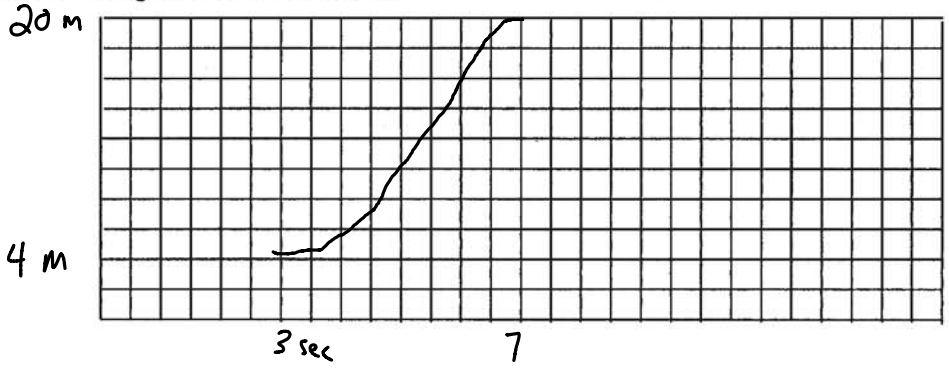
Sinusoidal axis - 18 - 8 = 10 (v-shift)

h-shift - max at t=0, no h-shift for cosine

$y = 10 + 8\sin\frac{\pi}{3}(t + \frac{3}{2})$

2. Mr. Keeping, disguised as Mathman, a costumed crime fighter, is swinging back and forth in front of the window to Ms. Gibbons's math class. At t = 3s, he is at one end of his swing and 4m from the window. At t = 7s, he is at the other end of his swing and 20m from the window.

   a) Sketch the curve. Use the distance from the window on the vertical axis and the time in seconds along the horizontal axis.



   b) What is the equation (in terms of sine and cosine), which represents Mathman's motion?

period = 8 seconds, v-shift = 20 - 8 = 12

ymax = 20, ymin 4 $\Rightarrow a = 8$

h-shift = 7

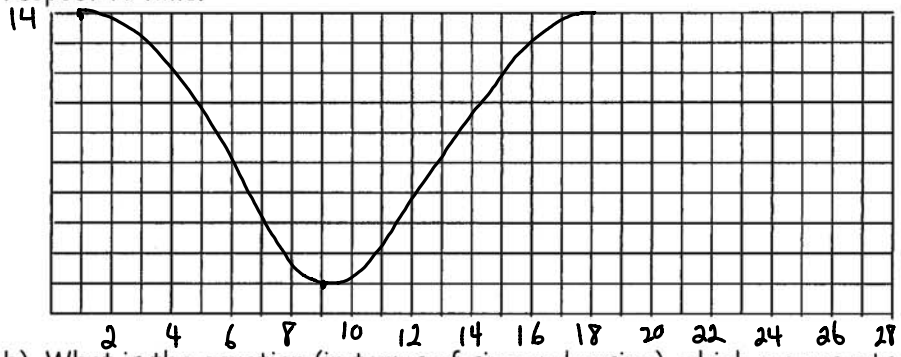
$y = 12 + 8\cos\frac{\pi}{4}(t - 7)$

$\frac{2\pi}{8}$ ← period = $\frac{\pi}{4}$

$y = 12 + 8\sin\frac{\pi}{4}(t - 4)$

3. John is floating on a tube in a wave tank. At t = 1second, John reaches a maximun height of 14m above the bottom of the pool. At t = 9seconds, John reaches a minimum height of 2m above the bottom of the pool

a) Sketch a graph below which expresses John's height from the bottom of the pool with respect to time.



b) What is the equation (in terms of sine and cosine), which represents John's motion?

max - 14, min - 2 $\Rightarrow$ $a = 12/2 = 6$

period = 16, $b = \frac{2\pi}{16} = \frac{\pi}{8}$

v-shift = $14 - 6 = 8$

h-shift = 1 sec ← for cosine

$y = 8 + 6\cos\frac{\pi}{8}(t-1)$

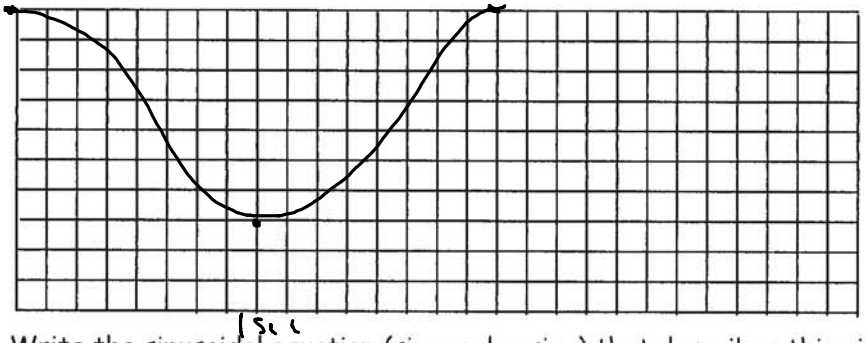
$y = 8 + 6\sin\frac{\pi}{8}(t+3)$

c) What is John's height from the bottom of the pool at 21 seconds?

$y(21) = 8$

4. A pendulum on a grandfather clock is swinging back and forth as it keeps time. A device is measuring the distance the pendulum is above the floor as it swings back and forth. At the beginning of the measurements the pendulum is at its highest point, 36cm high exactly one second later it was at its lowest point of 12cm. One second later it was back to its highest position.

a) Use the information above to sketch a diagram of this sinusoidal movement.



b) Write the sinusoidal equation (sine and cosine) that describes this situation.

max = 36, min = 12 $\Rightarrow$ $24/2 = 12 = a$

period = 2, $b = \frac{2\pi}{2} = \pi$

v-shift = $36 - 12 = 24$

h-shift = 0 ← for cosine

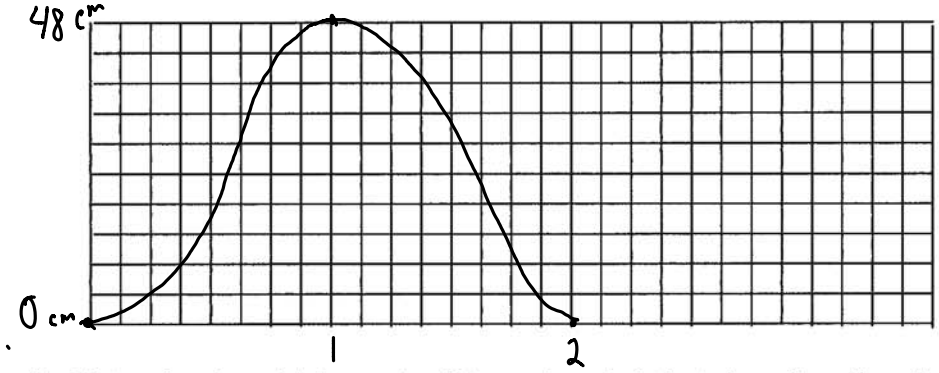
$y = 24 + 12\cos \pi t$

In terms of sine, h-shift is $\frac{1}{4}$ of the period to the left

$y = 24 + 12\sin\pi(t+\frac{1}{2})$

5. Sam is riding his bike home from school one day and picks up a nail in his tire. The nail hits the ground every 2 seconds and reaches a maximum height of 48 cm (assume the tire does not deflate).

a) Use the information above to sketch a diagram of this sinusoidal movement.



b) Write the sinusoidal equation (sine and cosine) that describes the situation in part a.

Max = 48, Min = 0 } 48/2 = 24

period = 2

$b = \frac{2\pi}{2} = \pi$

v-shift = 24

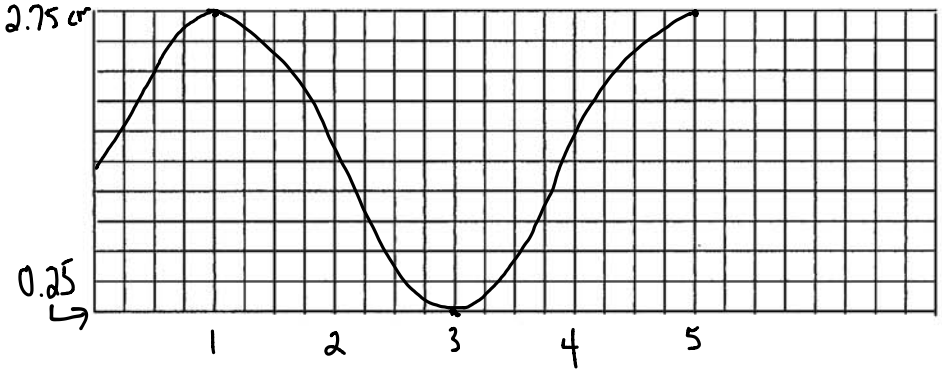
h-shift = 1 sec ← for cosine

$y = 24 + 24\cos \pi(t-1)$

$y = 24 + 24\sin \pi\left(t-\frac{1}{2}\right)$

6. Jackie, Nicolle and Maegan are playing skip rope. As the rope rotates it is observed that its maximum height is 2.75m after 1 second. The first minimum height of 0.25m occurs 2 seconds after the maximum height.

a) Use the information above to sketch a diagram of this sinusoidal movement.



b) Write the sinusoidal equation (sine and cosine) that describes the situation in part a.

Max = 2.75, Min = 0.25 } $a = \frac{2.75 - 0.25}{2}$

period = 4

$b = \frac{2\pi}{4} = \frac{\pi}{2}$

v-shift = $\frac{2.75 + 0.25}{2}$

$= 1.5$

h-shift = 1 ← for cosine

$y = 1.5 + 1.25\cos \frac{\pi}{2}(t-1)$

$y = 1.5 + 1.25\sin \frac{\pi}{2}(t)$