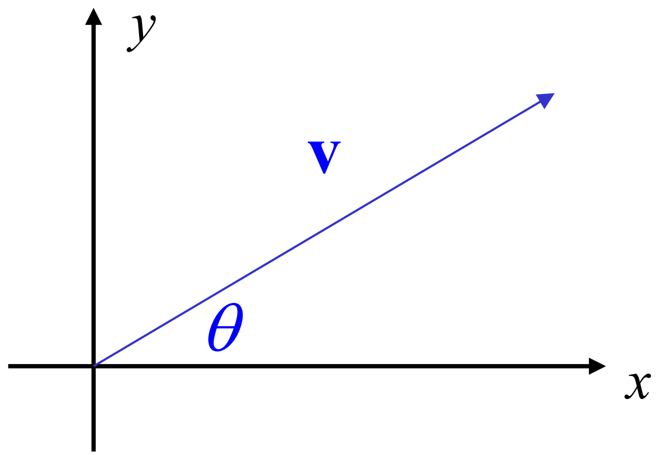
1-4 Vectors

Standard 1h: Find and draw a resultant vector from other component vectors.

Standard 1i: Find the direction angle of a resultant vector from other component vectors.

Standard 1j: Model and Solve problems involving vectors

A vector is in standard position if the initial point is at the origin.



What if we only knew the $\underbrace{\text{length}}_{\mathbf{v}}$ of the vector and the $\underbrace{\text{angle}}_{\theta}$?

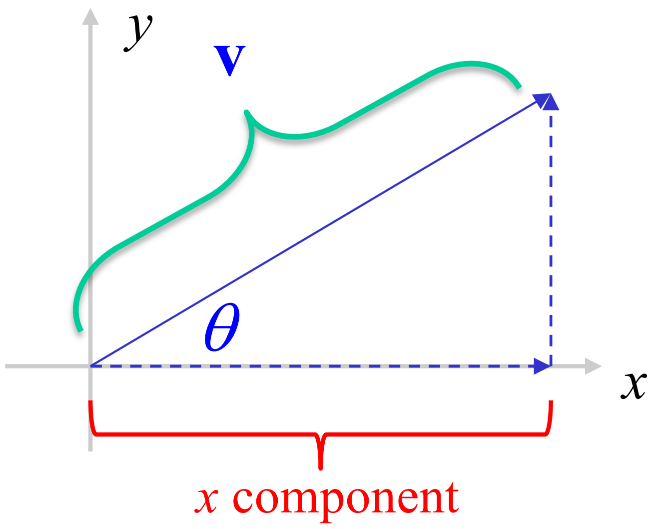
The component form of this vector is:

$$\vec{\mathbf{v}} = \left(\mathbf{v}\cos\theta\right)i + \left(\mathbf{v}\sin\theta\right)j \quad \text{or} \quad \vec{\mathbf{v}} = \left\langle \mathbf{v}\cos\theta, \mathbf{v}\sin\theta \right\rangle$$

Before anyone panics, this is just SOHCAHTOA…

Just watch…

A vector is in standard position if the initial point is at the origin.



$$\frac{adj}{hyp} = \frac{x\text{ component}}{\mathbf{v}} = \cos\theta$$

$$x\text{ component} = \mathbf{v}\cos\theta$$

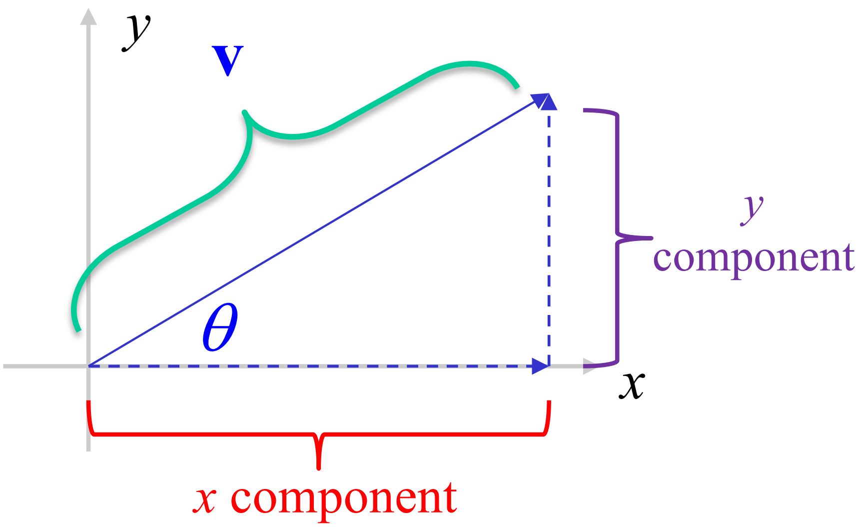
Remember what this really means: $\mathbf{v}$ ← Think of it as a hypotenuse of the right triangle above because it's the length of the vector

$$\vec{\mathbf{v}} = \underbrace{\left(\mathbf{v}\cos\theta\right)}_{x\text{ component}} i + \left(\mathbf{v}\sin\theta\right) j$$

or

$$\vec{\mathbf{v}} = \left\langle \underbrace{\mathbf{v}\cos\theta}_{x\text{ component}}, \mathbf{v}\sin\theta \right\rangle$$

A vector is in <u>standard position</u> if the initial point is at the origin.



$$\frac{opp}{hyp} = \frac{y \text{ component}}{\mathbf{v}} = \sin\theta$$

$$y \text{ component} = \mathbf{v}\sin\theta$$

See? Just SOHCAHTOA

Remember what this really means: $\mathbf{v}$ ← Think of it as a hypotenuse of the right triangle above because it's the length of the vector

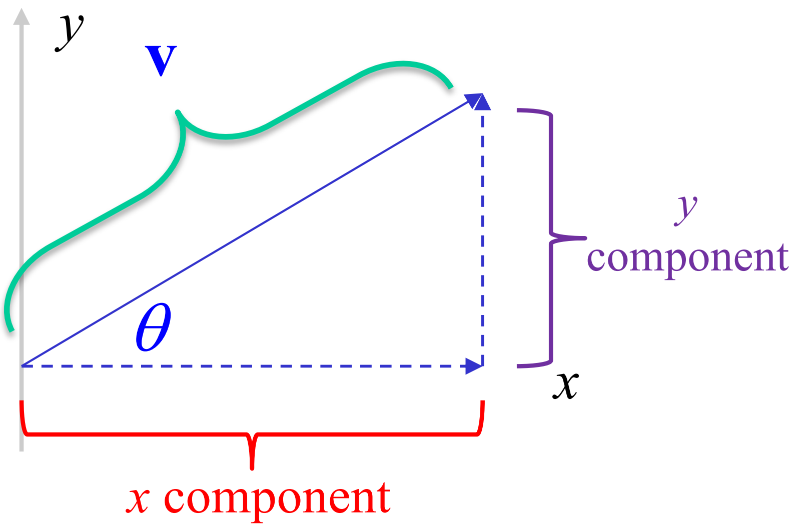
$$\vec{\mathbf{v}} = \underbrace{(\mathbf{v}\cos\theta)}_{x \text{ component}} i + \underbrace{(\mathbf{v}\sin\theta)}_{y \text{ component}} j$$

or

$$\vec{\mathbf{v}} = \left\langle \underbrace{\mathbf{v}\cos\theta}_{x \text{ component}}, \underbrace{\mathbf{v}\sin\theta}_{y \text{ component}} \right\rangle$$

If it’s the angle that you need to find, then you need to know this:

Remember that the magnitude and components form a right triangle



The direction of a vector **v** is found this way:

$$\cos\theta = \frac{adj}{hyp} = \frac{x\,\text{component}}{\mathbf{v}} = \frac{x_{\mathbf{v}}}{\mathbf{v}}$$

The direction **v** is the angle θ

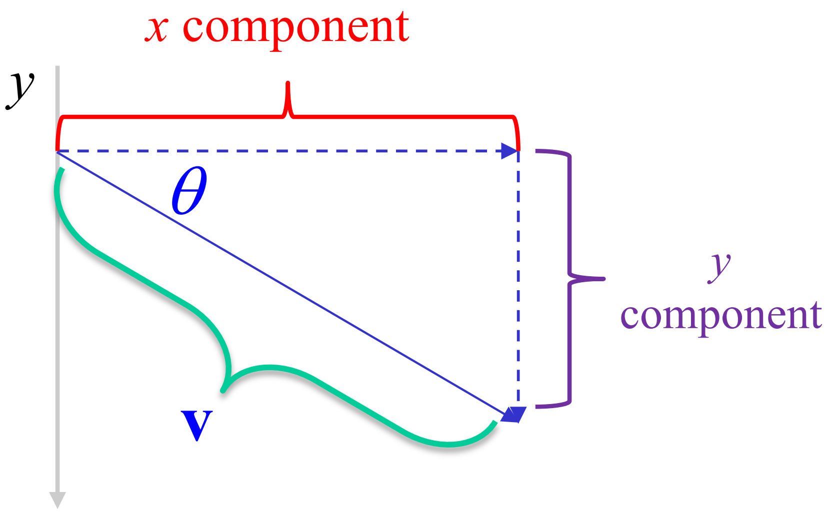
How would we determine which one?

$$\theta = \pm\cos^{-1}\left(\frac{x_{\mathbf{v}}}{\mathbf{v}}\right)$$

So this is just the *x* component divided by the magnitude

If it's the angle that you need to find, then you need to know this:

Remember that the magnitude and components form a right triangle



The direction of a vector **v** is found this way:

$$\cos\theta = \frac{adj}{hyp} = \frac{x\,\text{component}}{\mathbf{v}} = \frac{x_{\mathbf{v}}}{\mathbf{v}}$$

The direction angle here is negative because the *y* component is in a lower quadrant

The direction **v** is the angle *θ*

So the sign is determined by the sign of the *y* component

$$\theta = \pm\cos^{-1}\left(\frac{x_{\mathbf{v}}}{\mathbf{v}}\right)$$

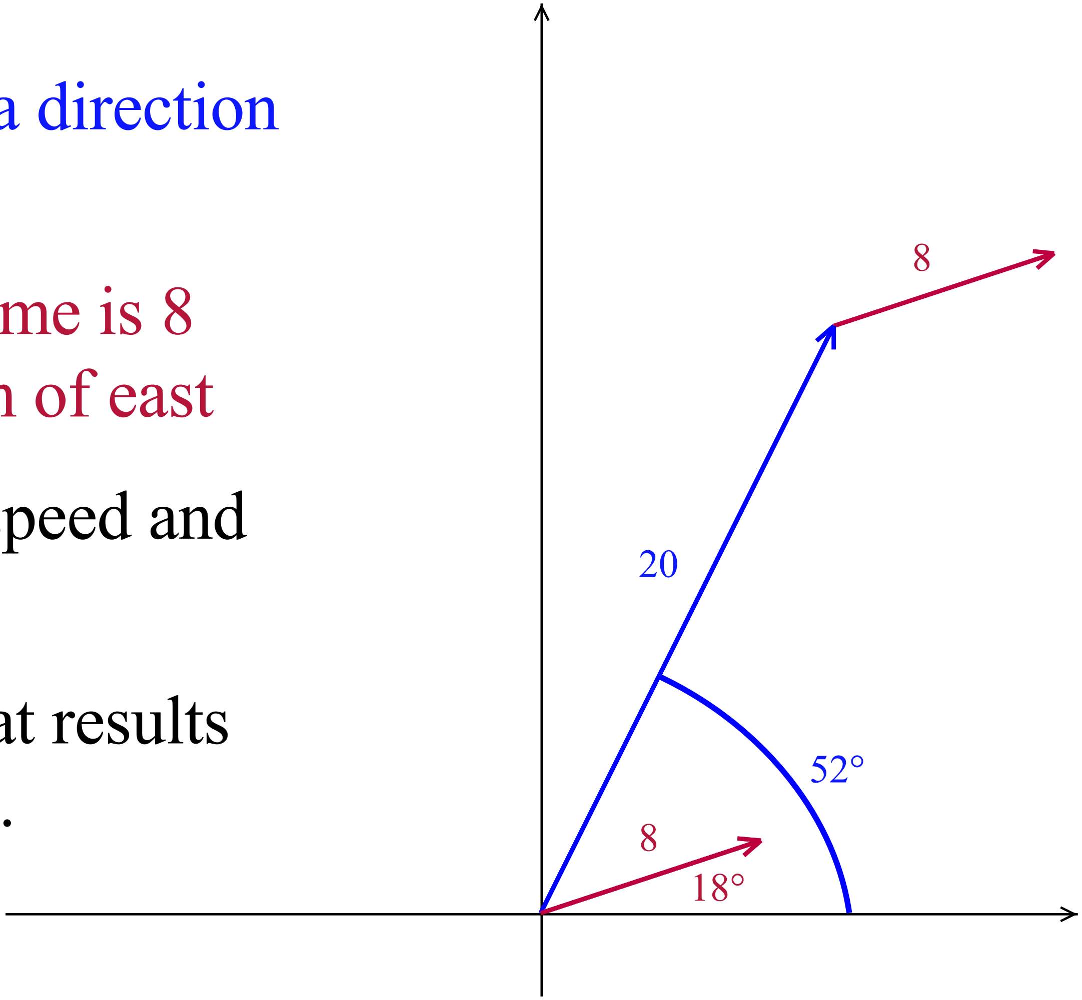
So this is just the *x* component divided by the magnitude

A boat is sailing at 20 mph with a direction of 52° north of east.

The current of the water at this time is 8 mph with a direction of 18° north of east

How does the current affect the speed and direction of the boat?

In other words find the vector that results from adding the first two vectors.



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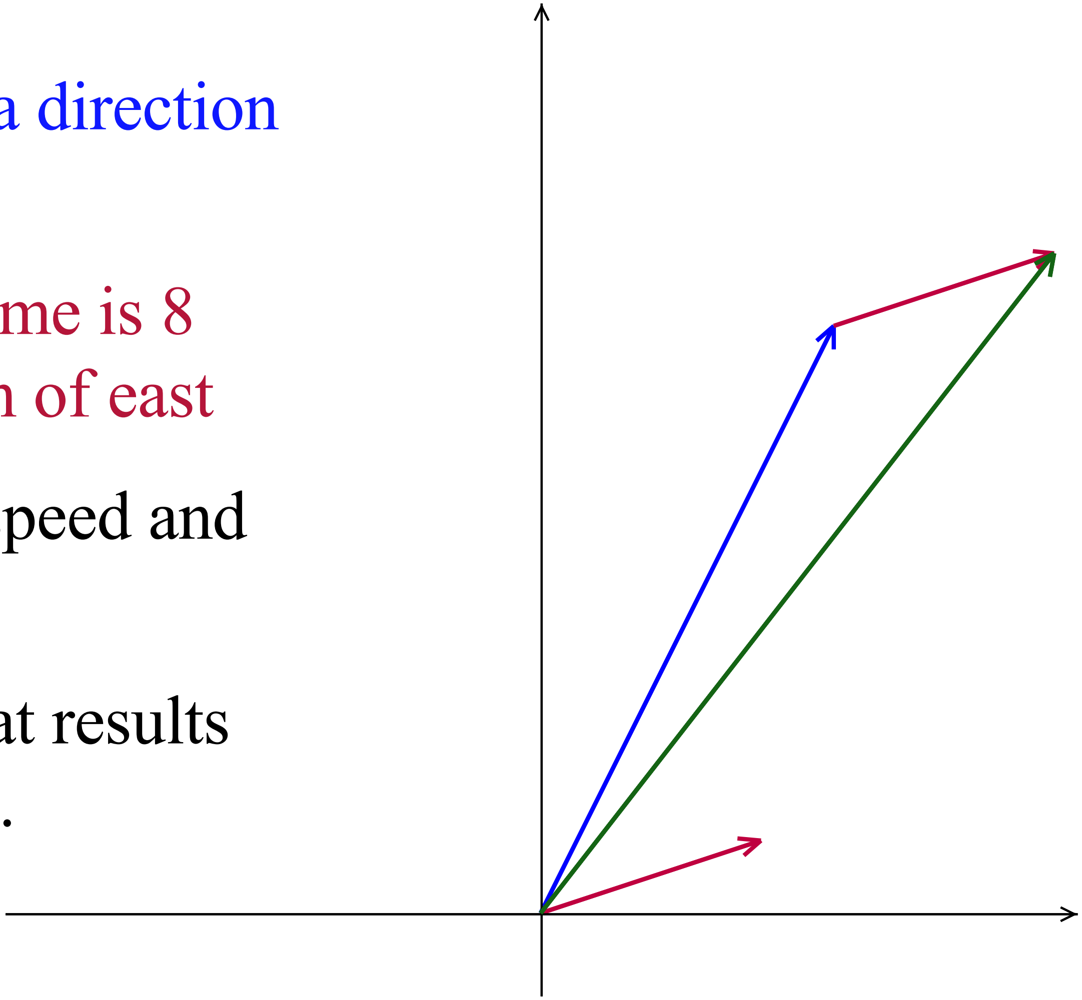


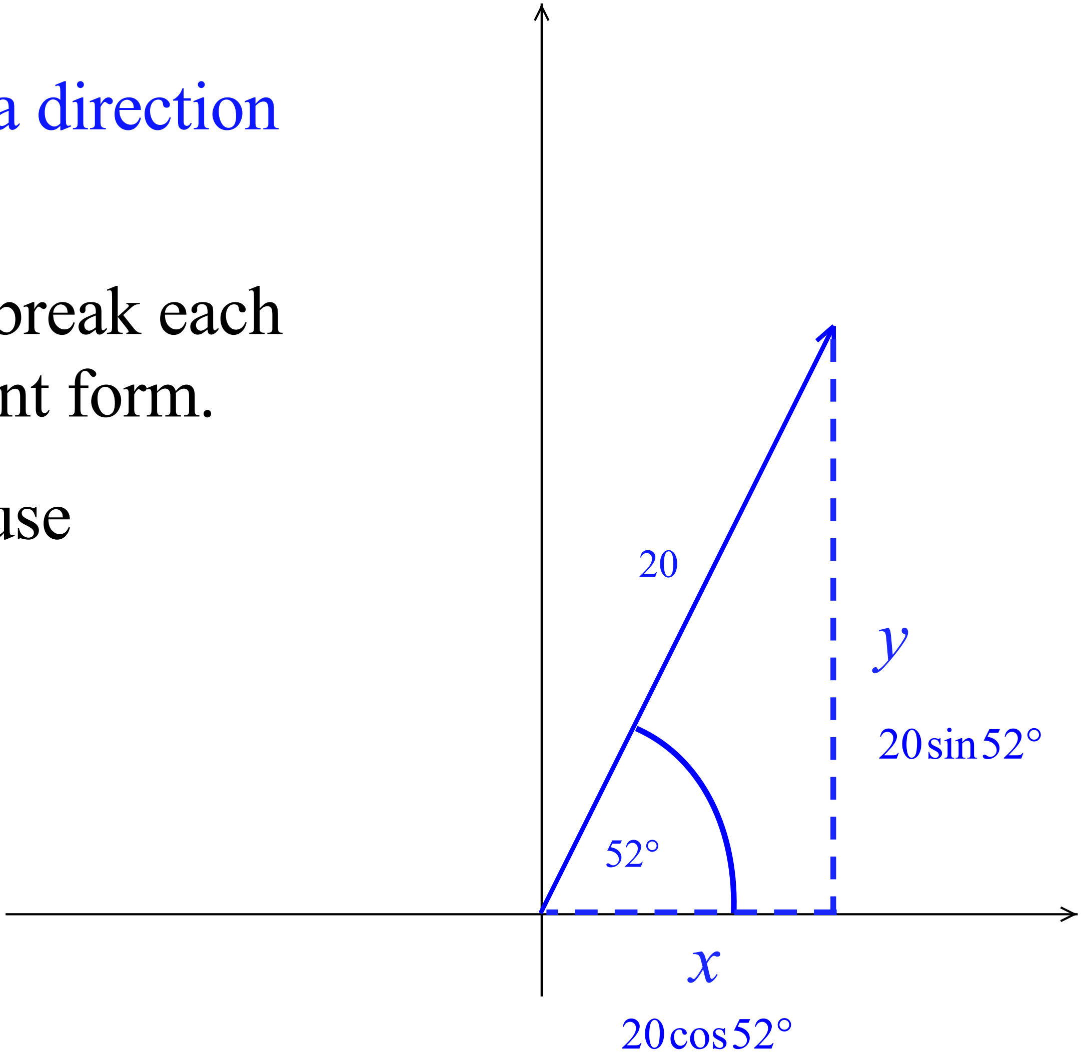
The sum of these two vectors will look like this.

In order to do add them we will have to break them into their x and y components one vector at a time.

A boat is sailing at 20 mph with a direction of 52° north of east.

In order to add them we need to break each down into their x and y component form.

Notice the right triangle so let's use SOHCAHTOA



$$\frac{y}{20} = \sin 52°$$

$$y = 20\sin 52°$$

$$\frac{x}{20} = \cos 52°$$

$$x = 20\cos 52°$$

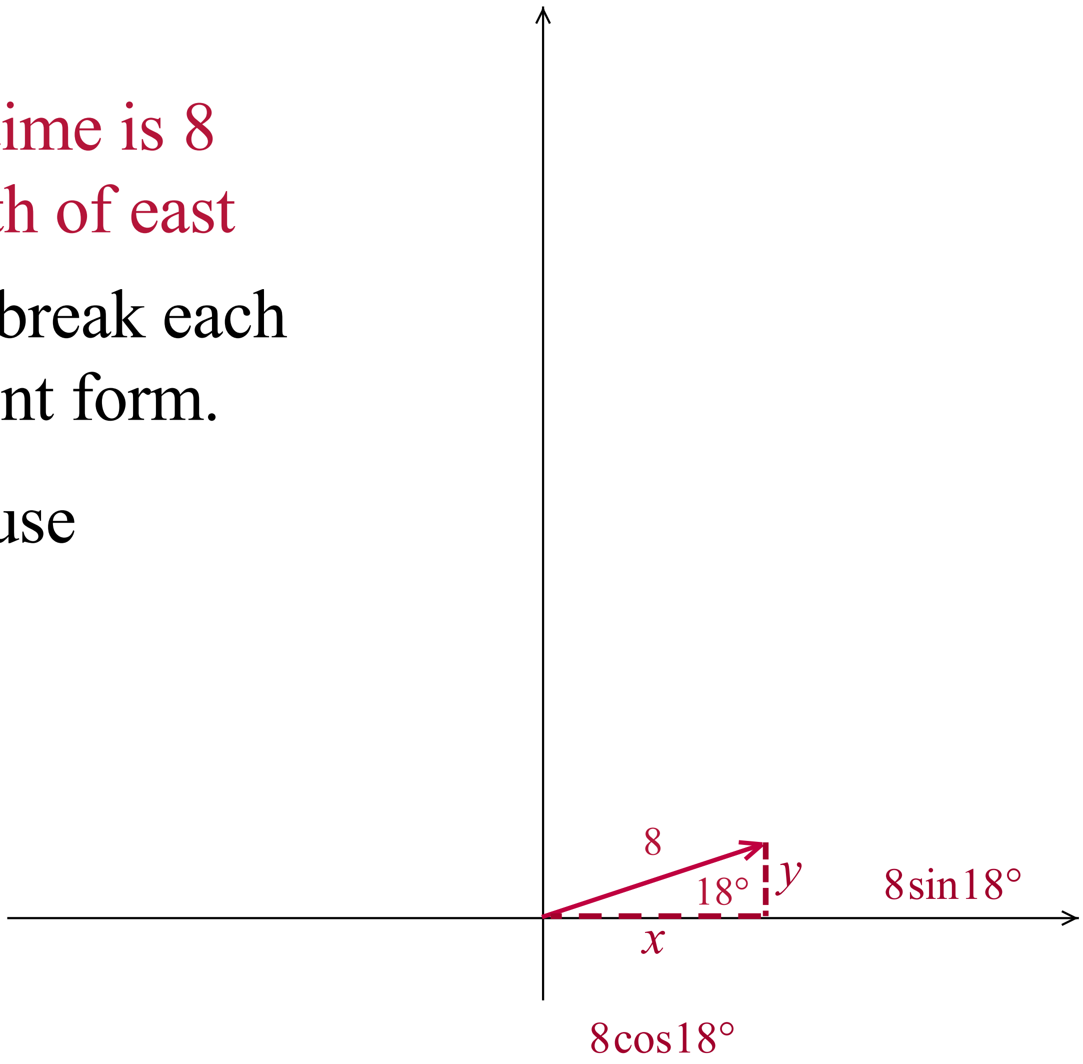
So the vector written in component form would look like this:

$$(20\cos 52°)i + (20\sin 52°)j$$

The current of the water at this time is 8 mph with a direction of 18° north of east

In order to add them we need to break each down into their x and y component form.

Notice the right triangle so let's use SOHCAHTOA



$$\frac{y}{8} = \sin 18°$$

$$y = 8\sin 18°$$

$$\frac{x}{8} = \cos 18°$$

$$x = 8\cos 18°$$

So the vector written in component form would look like this:

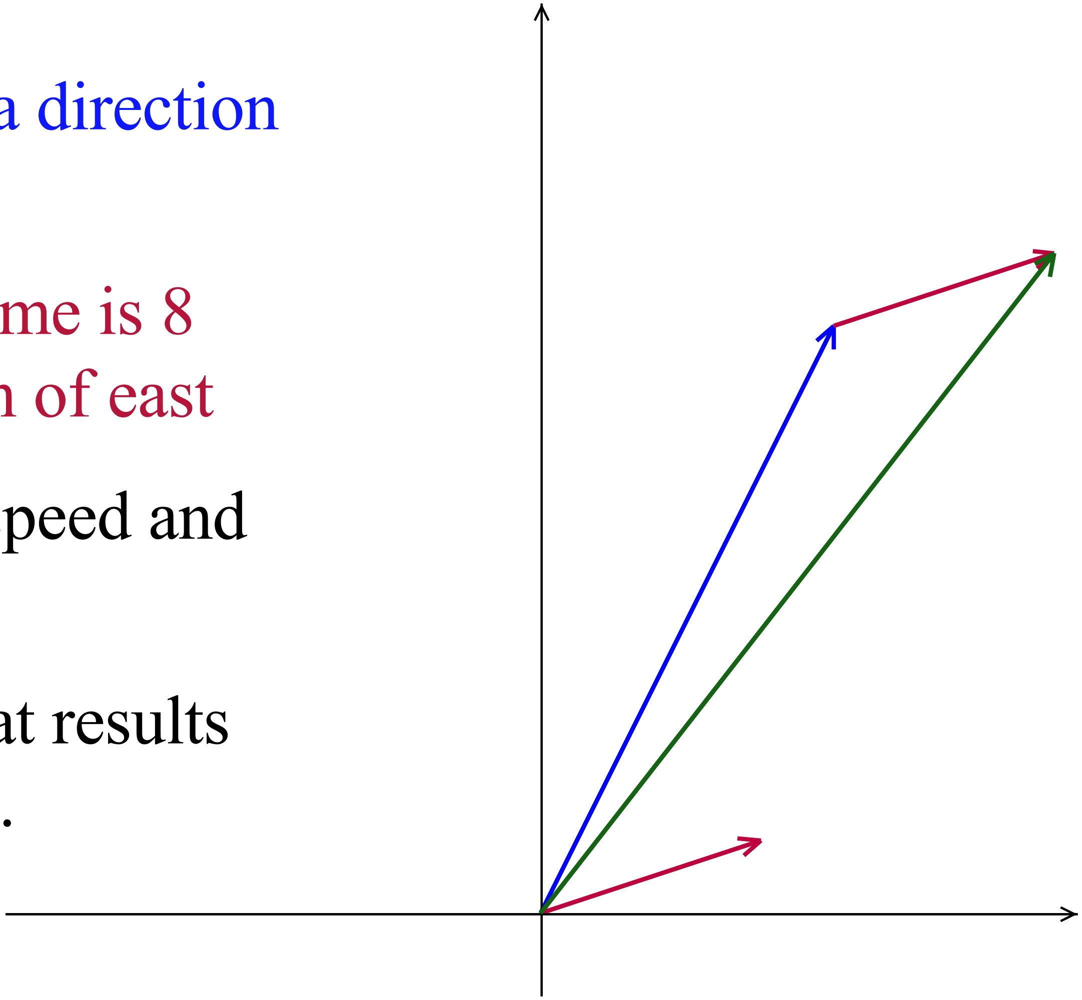
$$(8\cos 18°)i + (8\sin 18°)j$$

A boat is sailing at 20 mph with a direction of 52° north of east.

The current of the water at this time is 8 mph with a direction of 18° north of east

How does the current affect the speed and direction of the boat?

In other words find the vector that results from adding the first two vectors.



Now we just add their corresponding components

$$(8\cos 18°)i + (8\sin 18°)j$$

$$(20\cos 52°)i + (20\sin 52°)j$$

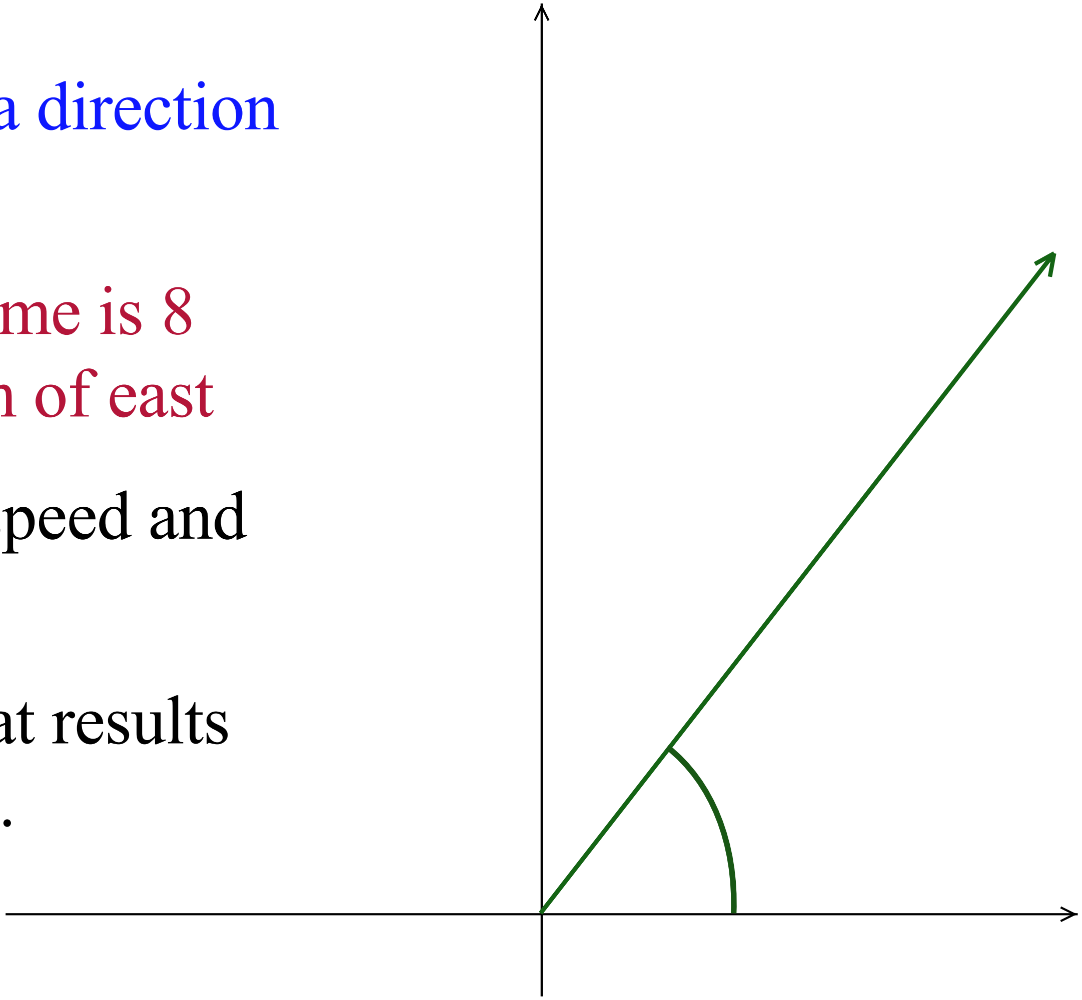
$$(20\cos 52° + 8\cos 18°)i + (20\sin 52° + 8\sin 18°)j$$

A boat is sailing at 20 mph with a direction of 52° north of east.

The current of the water at this time is 8 mph with a direction of 18° north of east

How does the current affect the speed and direction of the boat?

In other words find the vector that results from adding the first two vectors.

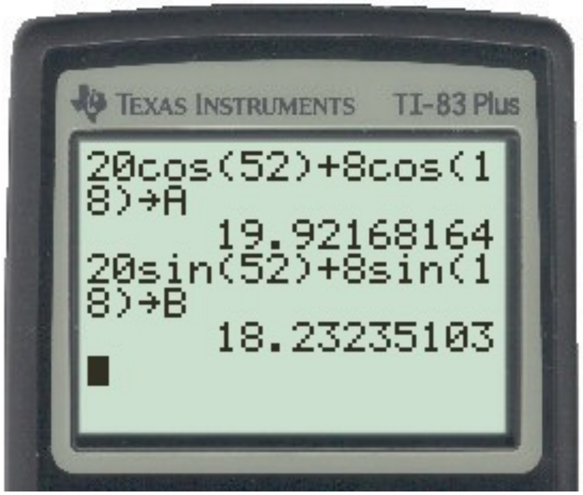


We'll use the calculator to find the actual values

$$(20\cos 52° + 8\cos 18°)i + (20\sin 52° + 8\sin 18°)j$$

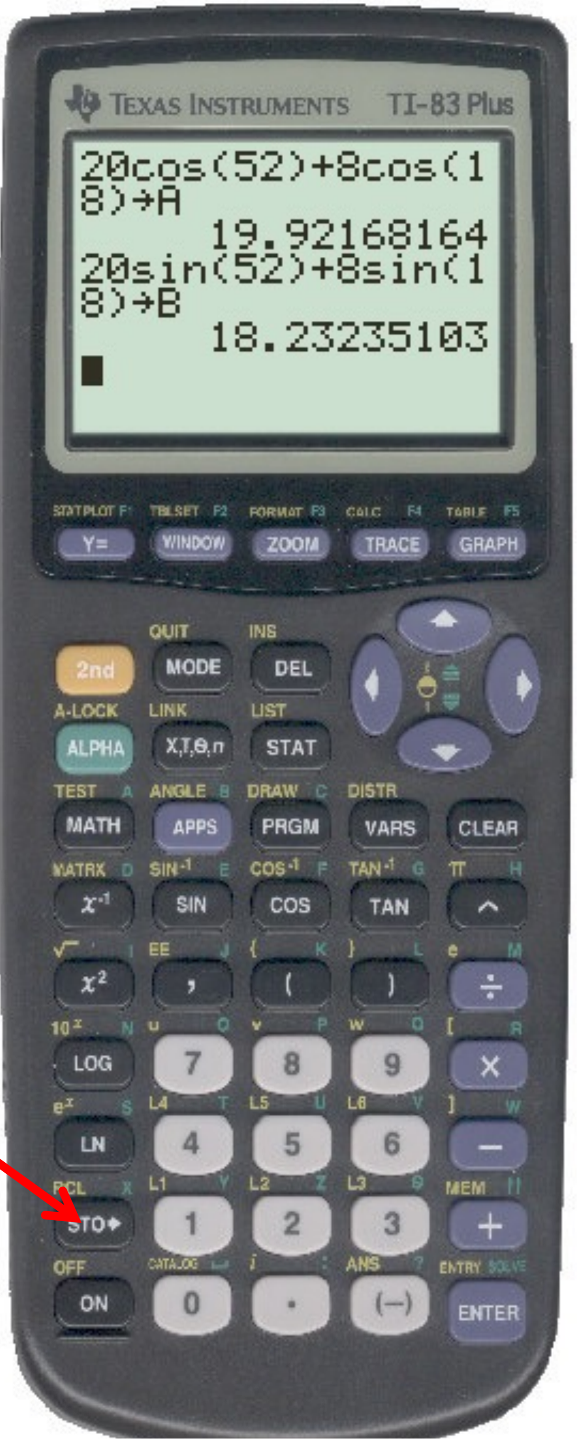
$$19.922i + 18.232j$$

Note that we stored the values which will be explained shortly



```
20cos(52)+8cos(1
8)→A
        19.92168164
20sin(52)+8sin(1
8)→B
        18.23235103
█
```

We’ll use the calculator to find the actual values we don’t want to round anything until we have our final answer.



```
20cos(52)+8cos(1
8)→A
      19.92168164
20sin(52)+8sin(1
8)→B
      18.23235103
```

Enter your calculations then press this button followed by any letter.

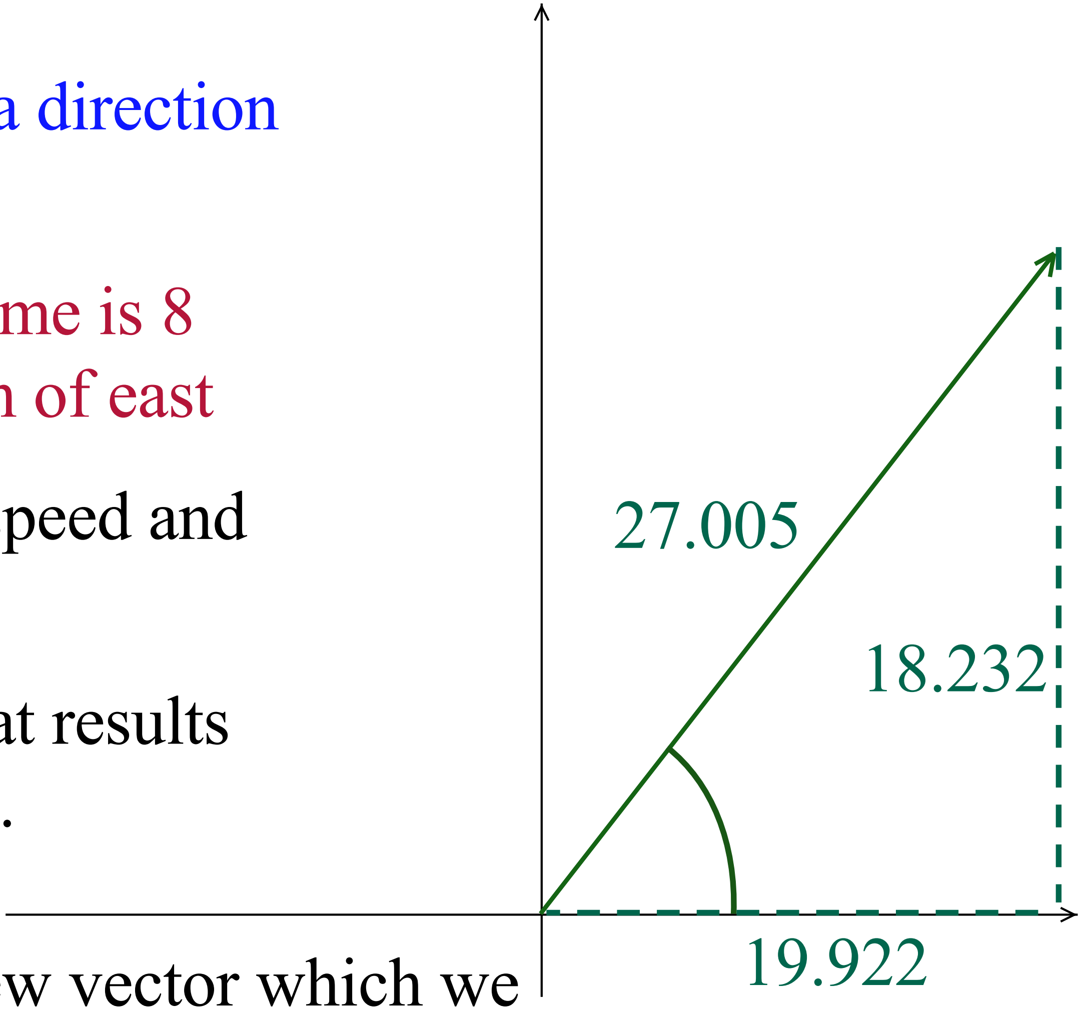
The calculator stores the exact values here in A and B so we can use them later.

A boat is sailing at 20 mph with a direction of 52° north of east.

The current of the water at this time is 8 mph with a direction of 18° north of east

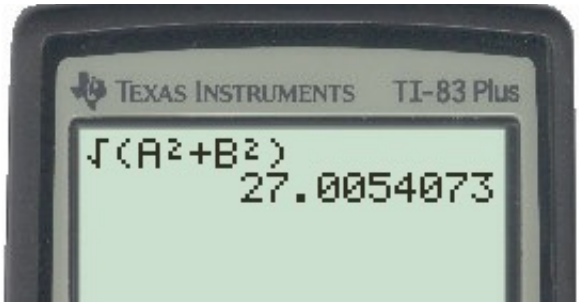
How does the current affect the speed and direction of the boat?

In other words find the vector that results from adding the first two vectors.



The speed is the length of this new vector which we can find using the Pythagorean Theorem and the stored values in the calculator.

27.005 mph



```
√(A²+B²)
               27.0054073
```

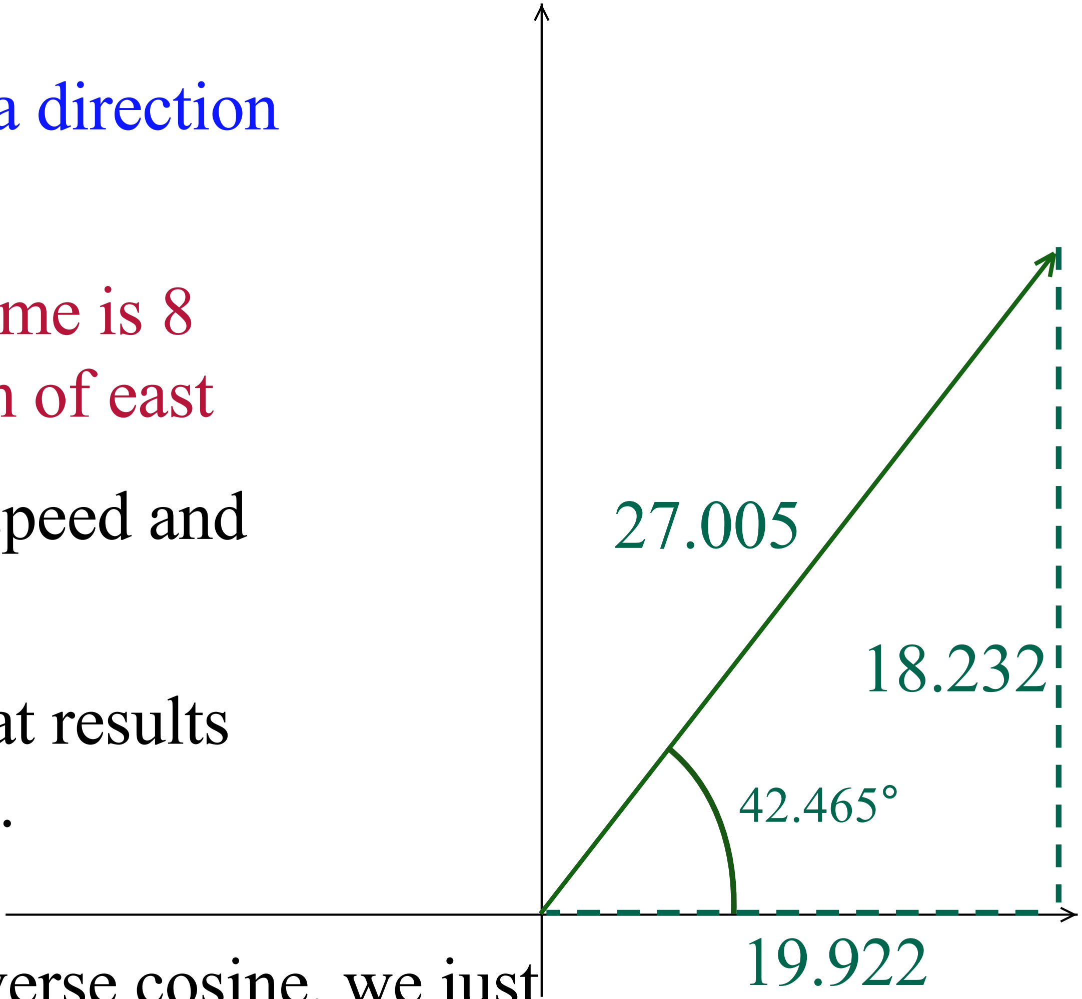
But what about it’s new direction?

A boat is sailing at 20 mph with a direction of 52° north of east.

The current of the water at this time is 8 mph with a direction of 18° north of east

How does the current affect the speed and direction of the boat?

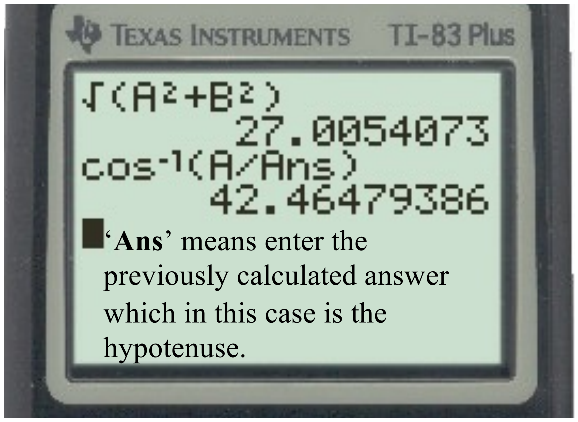
In other words find the vector that results from adding the first two vectors.



Remembering the slide about inverse cosine, we just take the x component (adjacent) and divide it by the speed we just found (hypotenuse)

42.465°

And we know it's positive because the y component is positive. We can also see this because it is in the first quadrant.



```
√(A²+B²)
            27.0054073
cos⁻¹(A/Ans)
           42.46479386
```

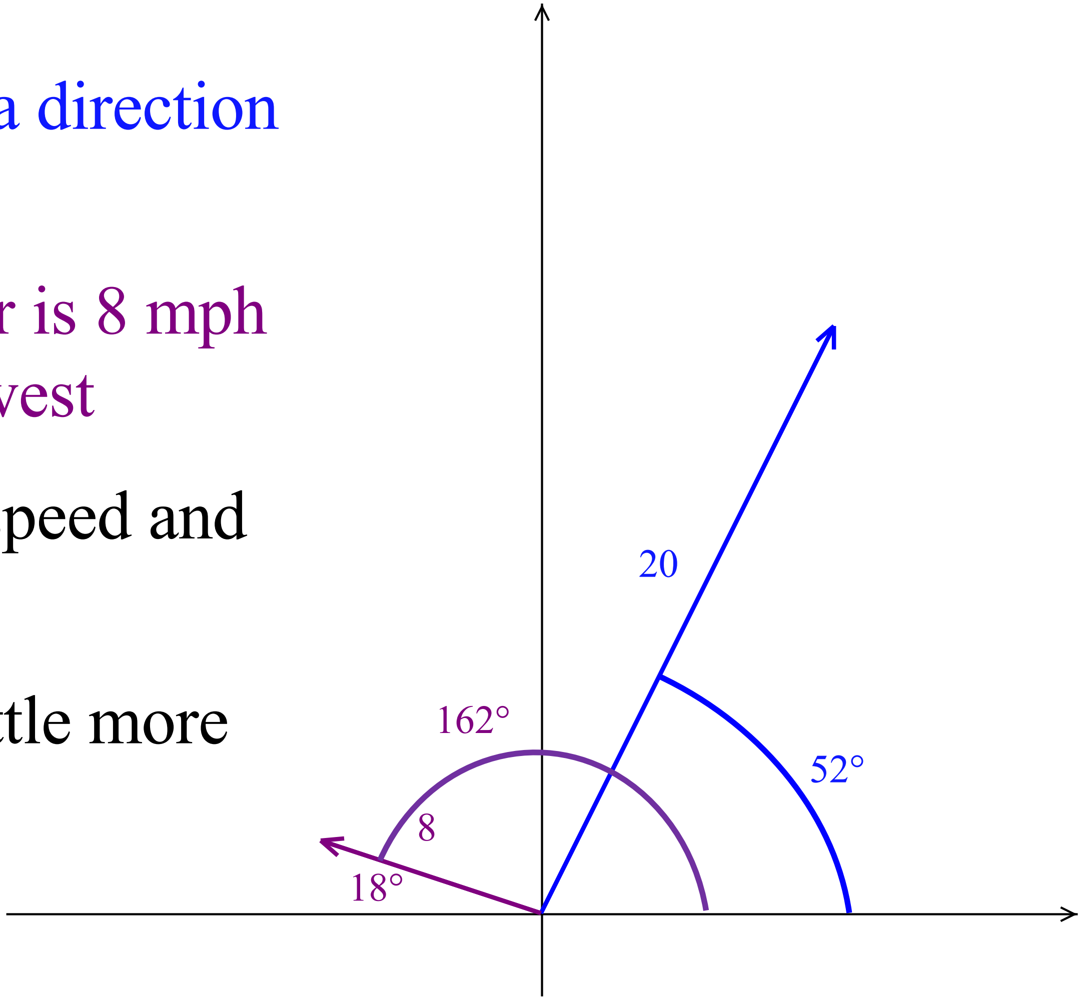
'Ans' means enter the previously calculated answer which in this case is the hypotenuse.

A boat is sailing at 20 mph with a direction of 52° north of east.

This time the current of the water is 8 mph with a direction of 18° north of west

How does the current affect the speed and direction of the boat?

This time we will have to do a little more work with the direction angles?



$$(20\cos 52°)i + (20\sin 52°)j$$

$$(8\cos 162°)i + (8\sin 162°)j$$

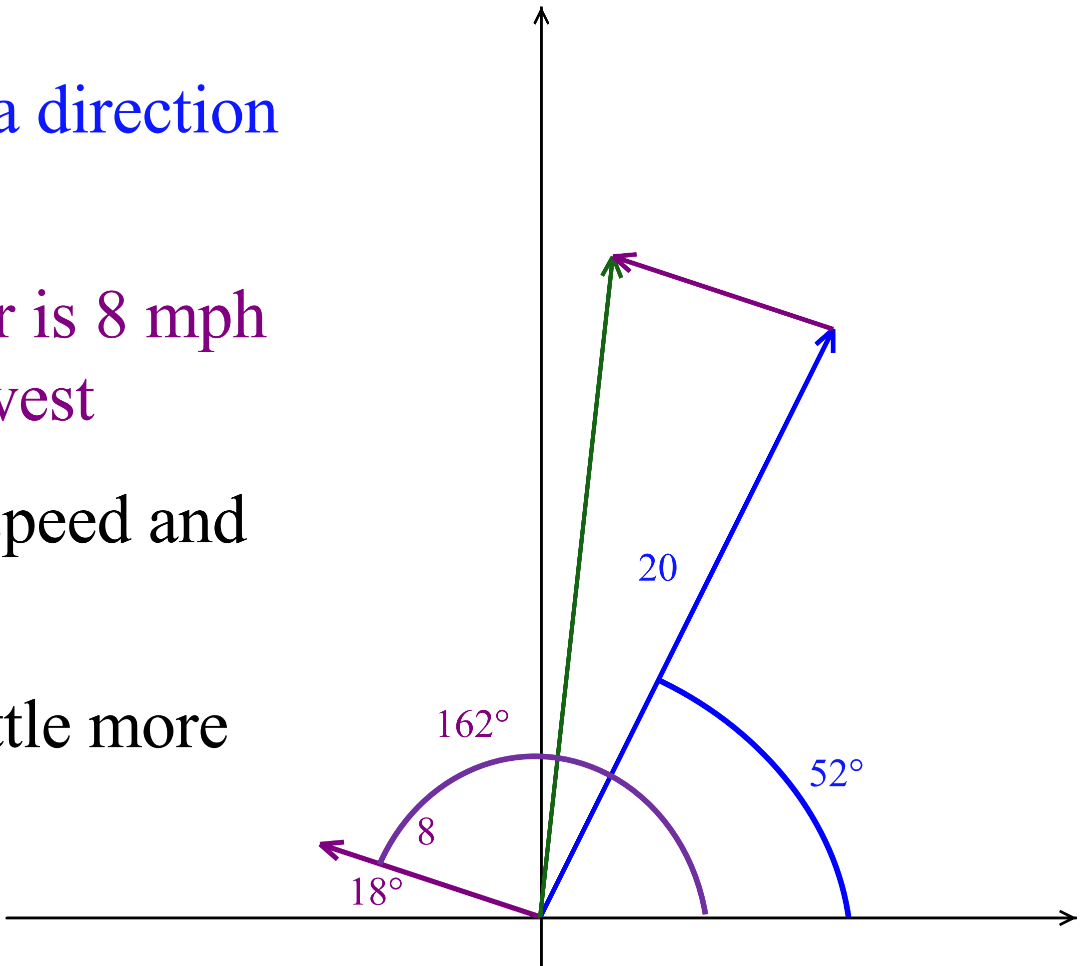
$$(20\cos 52° + 8\cos 162°)i + (20\sin 52° + 8\sin 162°)j$$

A boat is sailing at 20 mph with a direction of 52° north of east.

This time the current of the water is 8 mph with a direction of 18° north of west

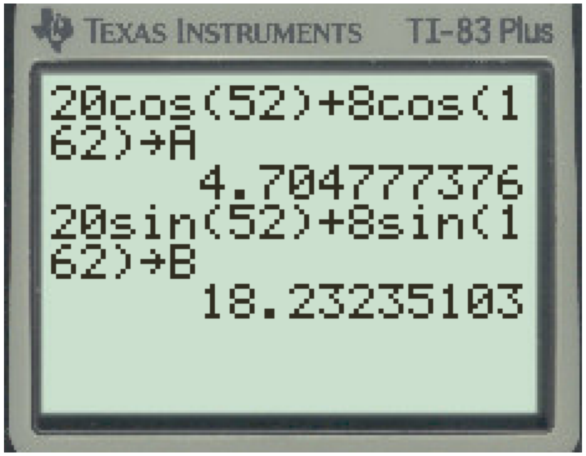
How does the current affect the speed and direction of the boat?

This time we will have to do a little more work with the direction angles?



$$(20\cos 52^\circ + 8\cos 162^\circ)i + (20\sin 52^\circ + 8\sin 162^\circ)j$$

$$4.705i + 18.232j$$



```
20cos(52)+8cos(1
62)→A
     4.704777376
20sin(52)+8sin(1
62)→B
     18.23235103
```

A boat is sailing at 20 mph with a direction of 52° north of east.

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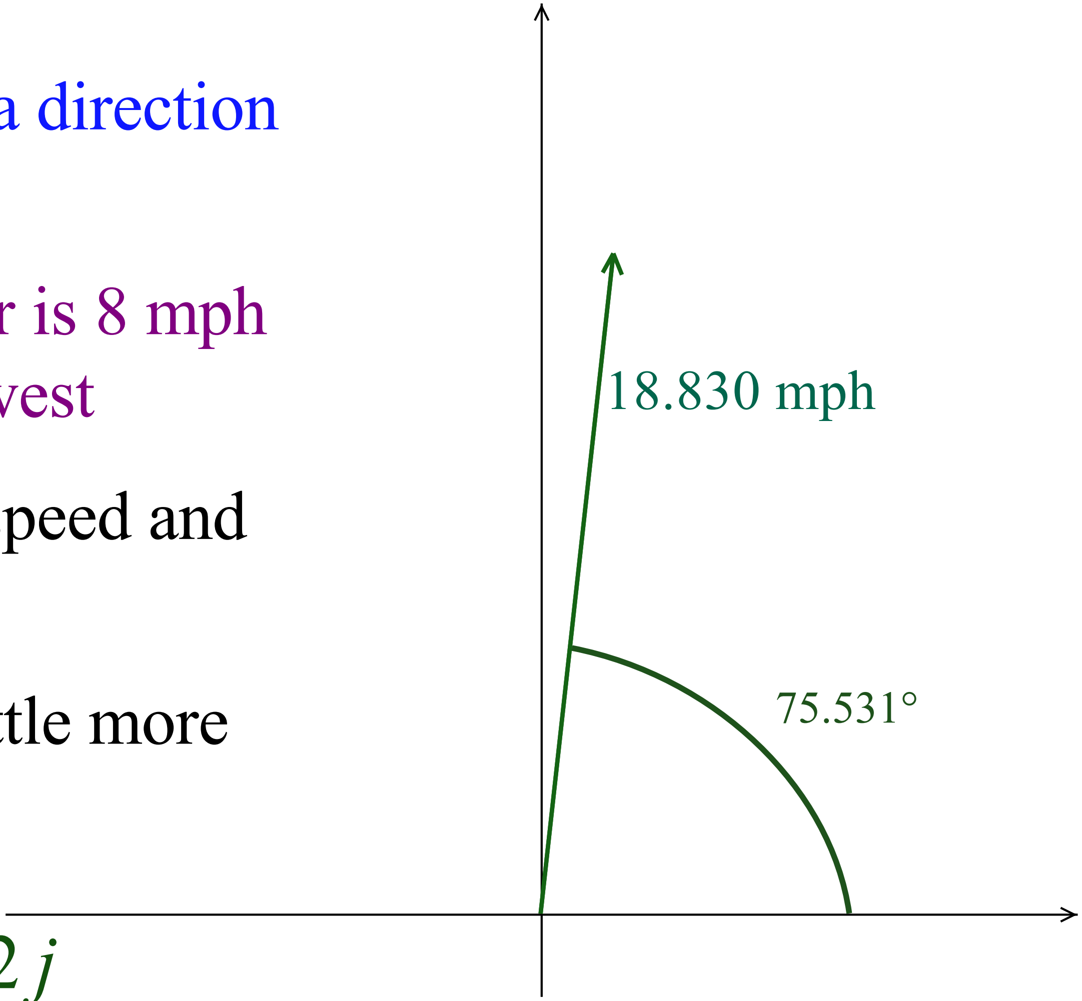
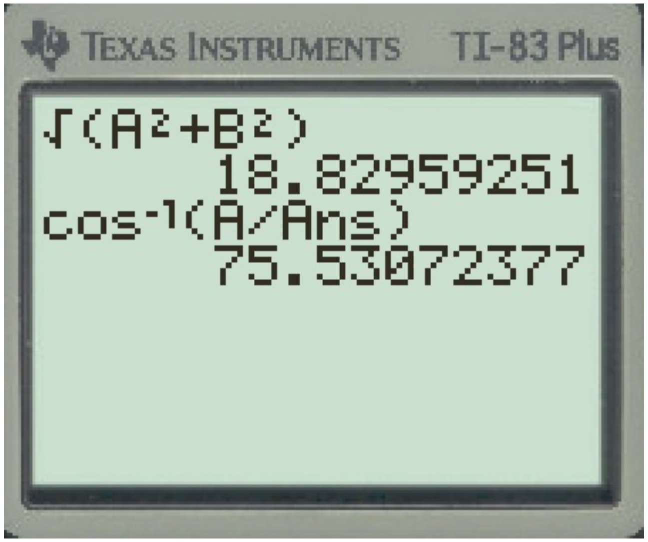


$$4.705i + 18.232j$$

$$\sqrt{(4.705)^2 + (18.232)^2} \approx 18.830$$

$$\cos^{-1}\left(\frac{4.705}{\sqrt{(4.705)^2 + (18.232)^2}}\right) \approx 75.531°$$



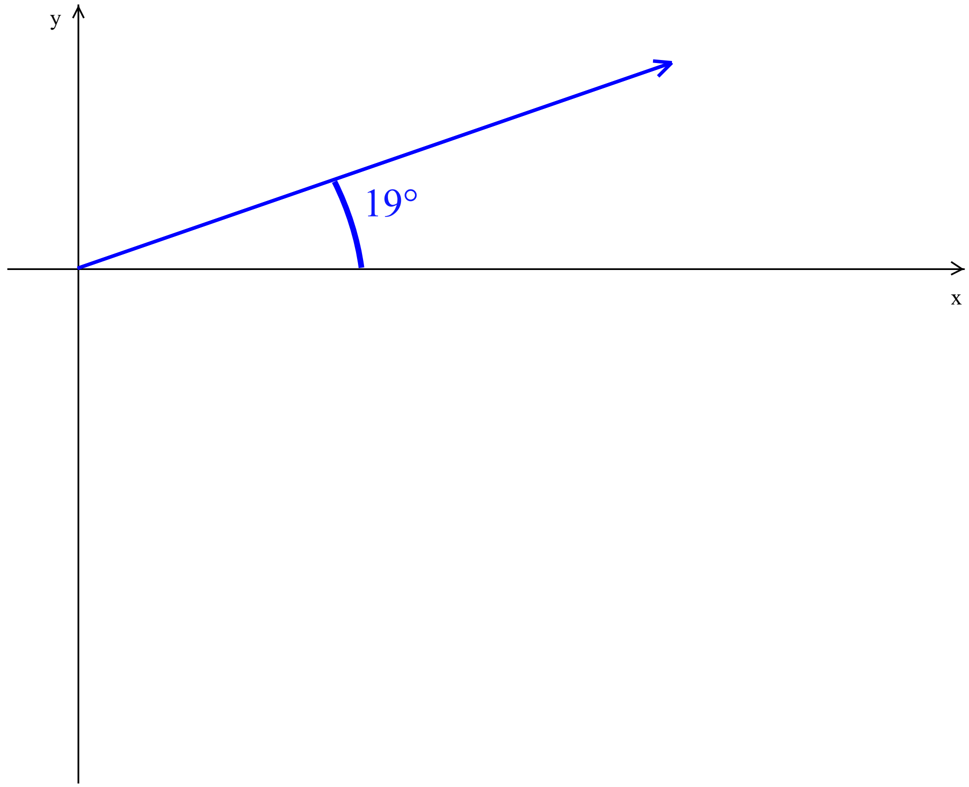
```
√(A²+B²)
        18.82959251
cos⁻¹(A/Ans)
        75.53072377
```

Now the boat is sailing at 20 mph with a direction of 19° north of east.

This time the current of the water is 17 mph with a direction of –68° (south of east)

How does the current affect the speed and direction of the boat?

$$(20\cos 19^\circ)i + (20\sin 19^\circ)j$$



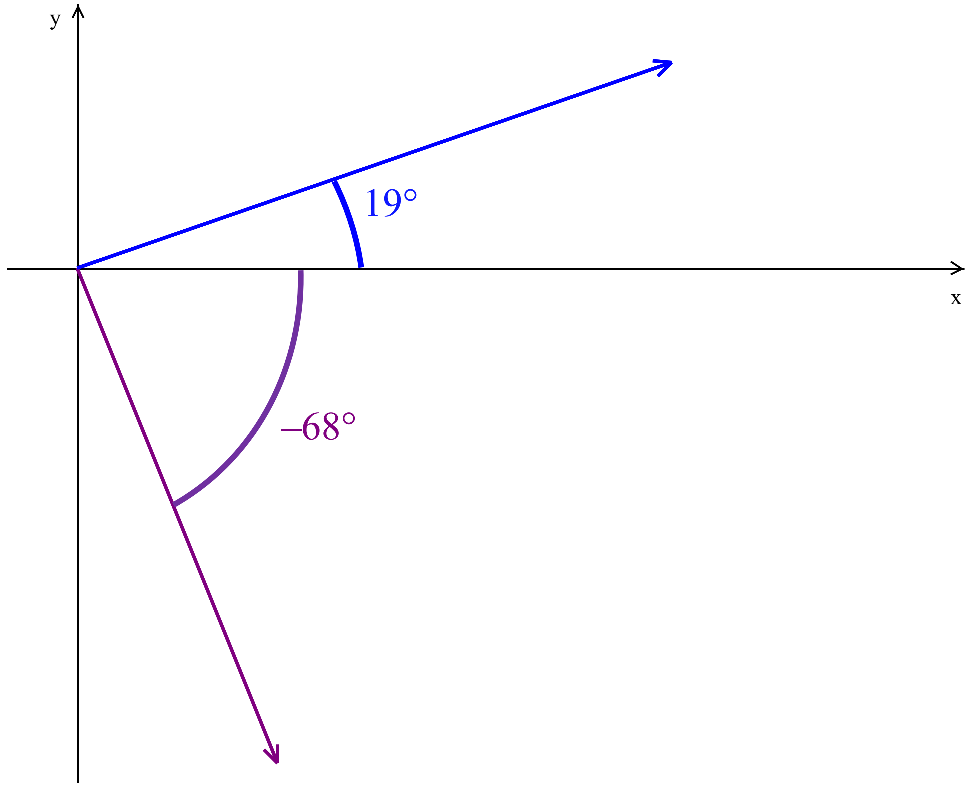
Now the boat is sailing at 20 mph with a direction of 19° north of east.

This time the current of the water is 17 mph with a direction of –68° (south of east)

How does the current affect the speed and direction of the boat?

$$(20\cos 19°)i + (20\sin 19°)\,j$$

$$(17\cos(-68°))i + (17\sin(-68°))\,j$$



Now the boat is sailing at 20 mph with a direction of 19° north of east.

This time the current of the water is 17 mph with a direction of –68° (south of east)

How does the current affect the speed and direction of the boat?

$$(20\cos 19° + 17\cos(-68°))i + (20\sin 19° + 17\sin(-68°))\,j$$

$$25.278...i - 9.251...\,j$$

$$\sqrt{(25.278...)^2 + (-9.251...)^2}$$

26.918 mph

$$\theta = \pm\cos^{-1}\left(\frac{25.278...}{\sqrt{(25.278...)^2 + (-9.251...)^2}}\right)$$

–20.100º