Determine if $x-7$ is a factor of $x^3 - x^2 + x - 1$

Watch this. Can you figure out what's going on?

$$\begin{array}{r|rrrr} 7 & 1 & -1 & 1 & -1 \\ & & 7 & 42 & 301 \\ \hline & 1 & 6 & 43 & 300 \end{array}$$

Determine if $x - 7$ is a factor of $1x^3 - 1x^2 + 1x - 1$

Let's try this again but slowly...

7	1	-1	1	-1
		7	42	301
	1	6	43	300

the coefficients of the polynomial

Start with the original coefficient

Multiply it by the number you're plugging in (7) and add the result to -1

This method is called <u>Synthetic Division</u>

And here's why...

Determine if $x - 7$ is a factor of $x^3 - x^2 + x - 1$

$$
\begin{array}{r|rrrr}
7 & 1 & -1 & 1 & -1 \\
 & & 7 & 42 & 301 \\
\hline
 & 1 & 6 & 43 & 300
\end{array}
$$

What this tells us is that if we try to factor x - 7, we get

$x^2 + 6x + 43$ With a remainder of 300

And where there's a remainder, it is not a factor

It is also called Synthetic Substitution because...

$$f(x) = x^3 - x^2 + x - 1$$

$$f(7) = 300$$

So we can find function values using this method which is why it is also called Synthetic Substitution

How many roots (zeros) does

$$y = x^3 - x^2 + x - 1$$

have?

In other words, what value(s) for x will give us $y = 0$ when we plug them in?

Here is where Synthetic Substitution can help

$$0 = x^3 - x^2 + x - 1$$

After setting y = 0, we could use grouping to factor this and get

$$0 = (x - 1)(x^2 + 1)$$

We could also use Synthetic Substitution since plugging in 1 would be so easy to do

We are going to choose 1 as our root since it is very easy to check if this is a good guess

Remember...

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 1 & -1 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

Start with the original coefficient

Multiply by 1 and add to the next coefficient

Repeat the process and if your remainder is 0, then (x – 1) is a factor

$$y = x^3 - x^2 + x - 1$$

$$y = (x - 1)(x^2 + 1)$$

Notice something about this term

$$\begin{array}{c|cccc} 1 & 1 & -1 & 1 & -1 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$y = (x - 1)(x^2 + 0x + 1)$$

What remains is in descending order beginning with a degree of 2 (one smaller than what we started with)

Still not sure?

Let's do another one then...

$$y = 1x^3 + 1x^2 - 4x - 4$$

1	1	1	−4	−4
		1	2	−2
	1	2	−2	$-6 \neq 0$

Which means what?

That 1 is not a root so $(x - 1)$ is not a factor

It does however mean that
$(1, -6)$ is a point on the graph of $y = x^3 + x^2 - 4x - 4$

Still not sure?

Let's go back and try −1

$$y = x^3 + x^2 - 4x - 4$$

−1	1	1	−4	−4
		−1	0	4
	1	0	−4	0

Which means what?

That −1 is a root so
(x + 1) is a factor

$$(x+1)(x^2+0x-4)$$

and so

$$x^3 + x^2 - 4x - 4 = (x+1)(x^2-4)$$

Since we now know that

$$x^3 + x^2 - 4x - 4 = (x+1)(x^2-4) = (x+1)(x-2)(x+2)$$

Let's go back and try 2 since we know it will work

$$y = x^3 + x^2 - 4x - 4$$

2	1	1	−4	−4
	—	2	6	4
	1	3	2	0

Which means what?

That 2 is a root so (x − 2) is a factor

$$(x-2)(x^2+3x+2)$$

and that

$$x^3 + x^2 - 4x - 4 = (x-2)(x+2)(x+1)$$

Show that -2 is a root of $y = x^4 - 16$

-2	1	0	0	0	-16
		-2	4	-8	16
	1	-2	4	-8	0

$$y = (x+2)(x^3 - 2x^2 + 4x - 8)$$

Let's reduce this all the way

2	1	-2	4	-8
		2	0	8
	1	0	4	0

$$y = (x+2)(x-2)(x^2+4)$$