Sign Patterns

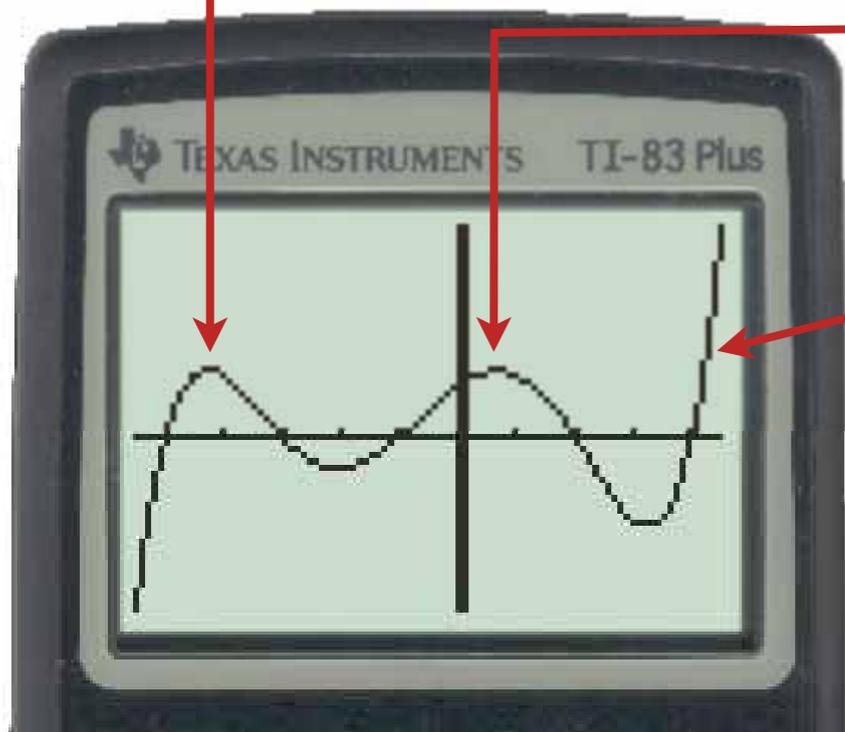$$y = x^5 + 3x^4 - 23x^3 - 51x^2 + 94x + 120$$

$$(x+1)(x-2)(x+5)(x-4)(x+3) > 0 \quad \text{when}$$

$$-5 < x < -3 \qquad \Rightarrow \quad x \in (-5, -3)$$

$$-1 < x < 2 \qquad \Rightarrow \quad x \in (-1, -2)$$

$$x > 4 \qquad \Rightarrow \quad x \in (4, \infty)$$

When is the graph of $y = x^2 + 4x - 5$

below the x-axis?

When is $y < 0$?

$x^2 + 4x - 5 < 0$ $(x + 5)(x - 1) < 0$

Which means that we want values of x for which

$(x + 5) < 0$ and $(x - 1) > 0$

or

$(x + 5) > 0$ and $(x - 1) < 0$

Two factors of opposite signs means the product will be negative

$(x + 5) < 0$ and $(x - 1) > 0$

$x < -5$ and $x > 1$

which never happens

or

$(x + 5) > 0$ and $(x - 1) < 0$

$x > -5$ and $x < 1$

$$\boxed{\begin{array}{c} -5 < x < 1 \\ \text{or} \\ x \in (-5, 1) \end{array}}$$

+   0   –   0   +

←————|————————————|——|————→
    −5   0 1

Now graph it

When is the graph of $y = x^3 + 5x^2 - x - 5$ above the x-axis?

When is $y > 0$?

$y = x^3 + 5x^2 - x - 5$

$y = x^2(x + 5) - (x + 5)$

$(x - 1)(x + 1)(x + 5) > 0$

$y = (x^2 - 1)(x + 5)$

$y = (x - 1)(x + 1)(x + 5)$

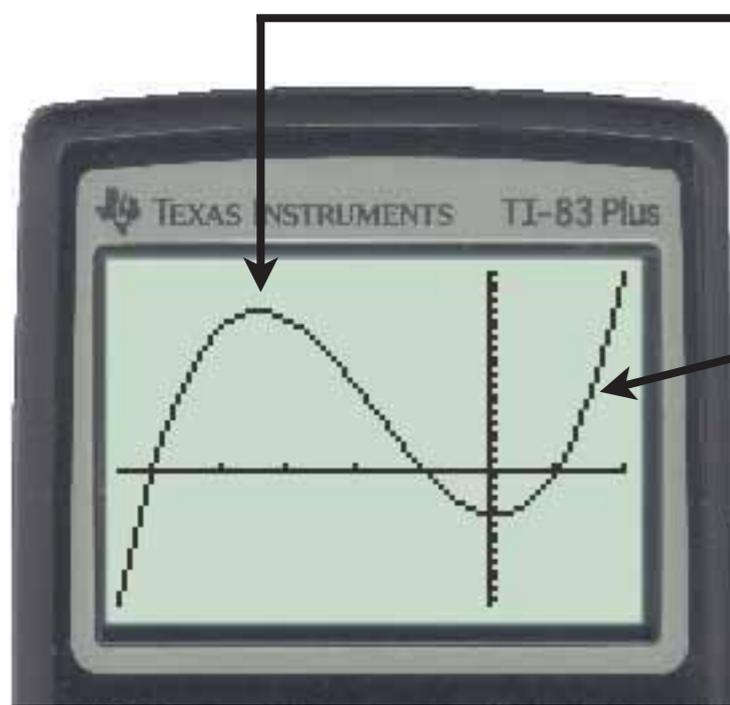When is the graph of $y = x^3 + 5x^2 - x - 5$

above the x-axis?

When is $y > 0$?

$(x - 1)(x + 1)(x + 5) > 0$

Which means that we want values of x for which

$$- \qquad 0 \qquad + \qquad 0 \quad - \quad 0 \qquad +$$

|←———————+———————————+———+————————→|
$$-5 \qquad\qquad -1 \qquad 1$$

$-5 < x < -1 \quad$ or $\quad x > 1$

$-5 < x < -1$

$x > 1$

$-5 < x < -1$ or $x > 1$

$x \in (-5, -1)$ ∪ $x \in (1, \infty)$