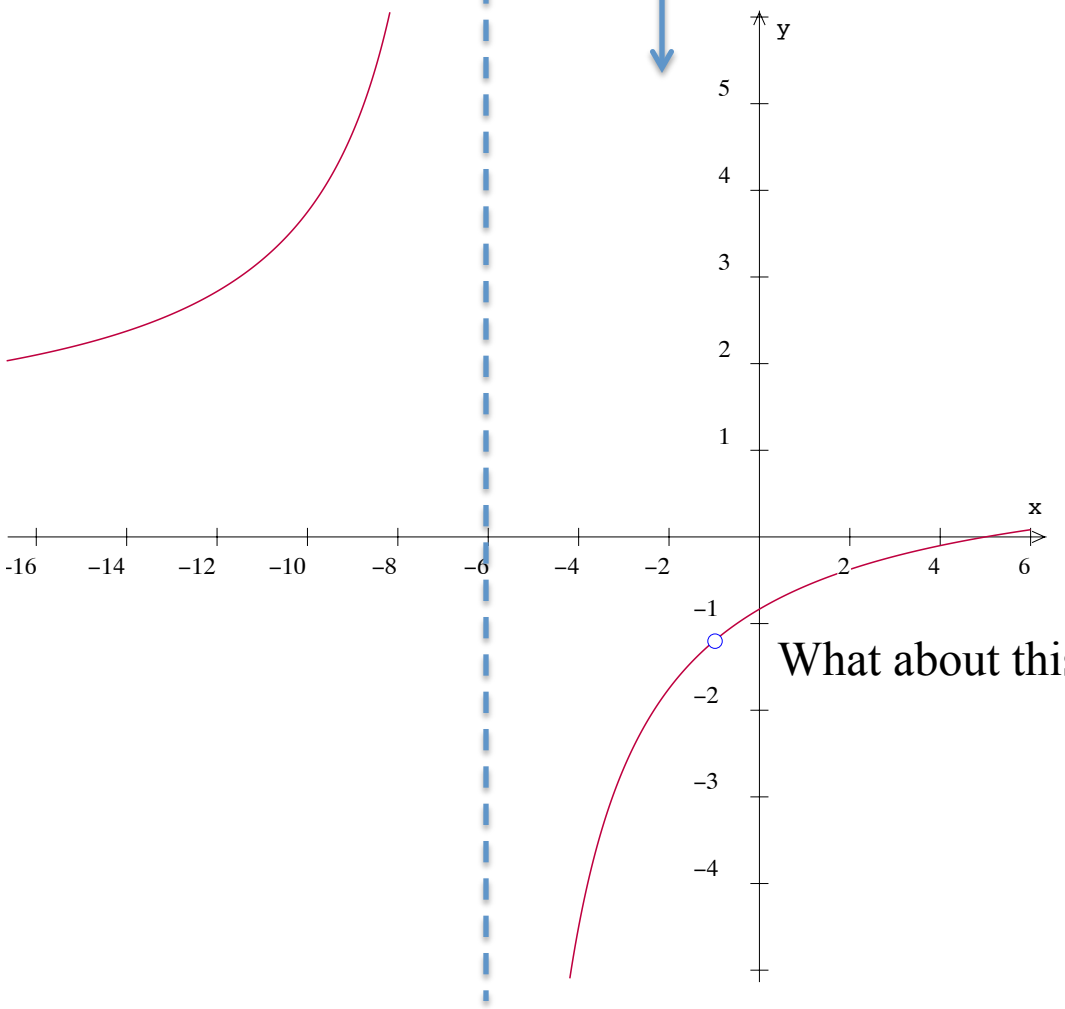
Rational Functions

Standard 4a

Find Zeros, Vertical Asymptotes, and Points of Exclusion of a Rational Function and distinguish them from one another

$$y = \frac{x^2 - 4x - 5}{x^2 + 7x + 6}$$

This is the graph of a this rational function?



What about this hole in the graph?

$$y = \frac{x^2 - 4x - 5}{x^2 + 7x + 6}$$

$$y = \frac{(x-5)(x+1)}{(x+6)(x+1)}$$

$$(x-5)(x+1) = 0$$

$$x = -1, 5$$

$$(x+6)(x+1) = 0$$

$$x = -1, -6$$

Zeros: Where $y = 0$ (x-intercepts)

Where only the numerator = 0

Vertical Asymptotes: Where y is undefined

Where only the denominator = 0

Points of Exclusion: (A hole in the graph): Where y is indeterminate

Where both the numerator and denominator = 0

$$x = -1$$

$$y = \frac{x^2 - 4x - 5}{x^2 + 7x + 6}$$

$$y = \frac{(x-5)(x+1)}{(x+6)(x+1)}$$

Where the numerator = 0

$$(x-5)(x+1) = 0$$

$$x = -1, 5$$

Where the denominator = 0

$$(x+6)(x+1) = 0$$

$$x = -1, -6$$

Zeros: Where $y = 0$ (x-intercepts)

Vertical Asymptotes: Where y is undefined

Points of Exclusion: (A hole in the graph): Where y is indeterminate

Where both the numerator and denominator = 0

$$x = -1$$

Zero: at $x = 5$

Vertical Asymptote: at $x = -6$

Point of Exclusion: at $x = -1$

$$y = \frac{x^2 - 4x - 5}{x^2 + 7x + 6}$$

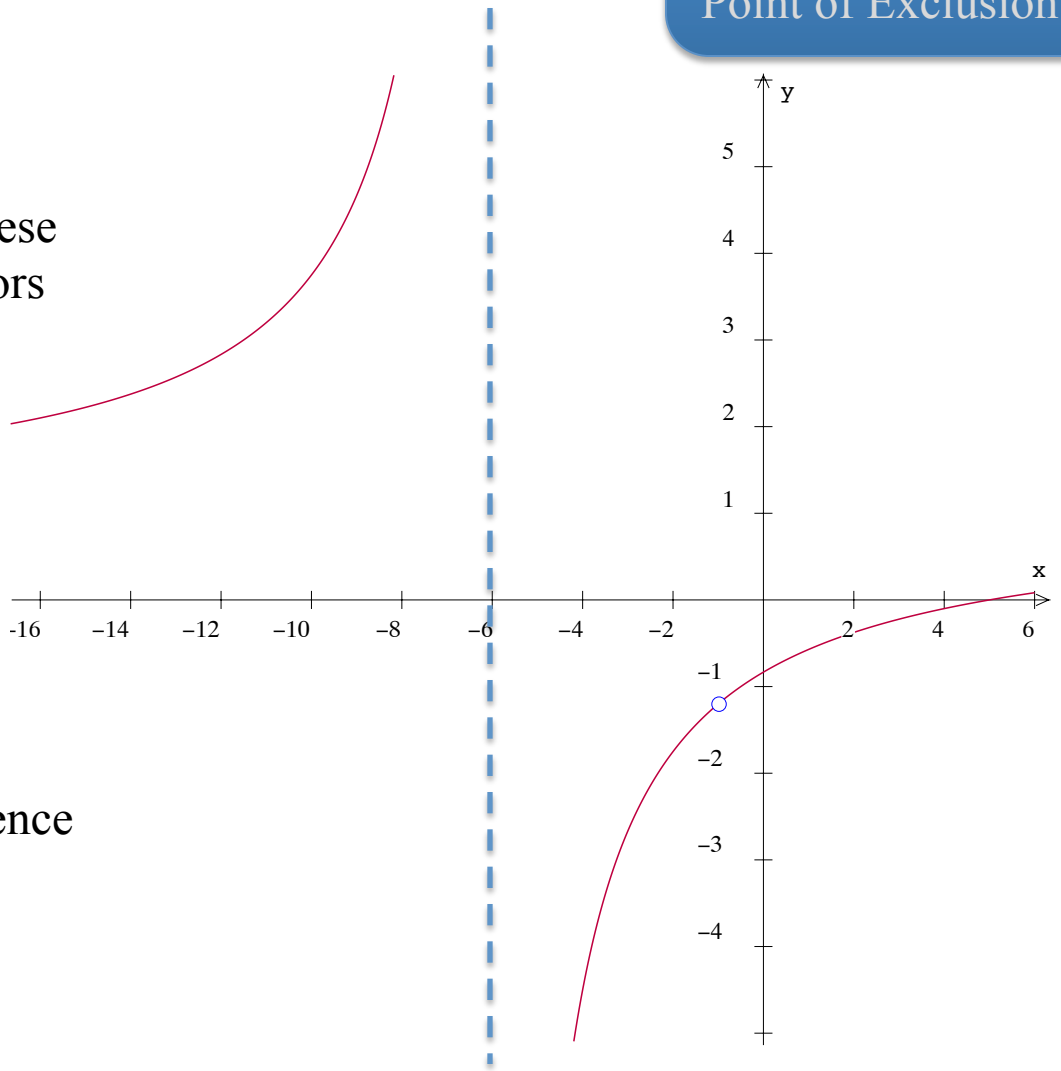
Zero: at $x = 5$

Vertical Asymptote: at $x = -6$

Point of Exclusion: at $x = -1$

Graph both of these on your calculators

$$y = \frac{(x-5)}{(x+6)}$$



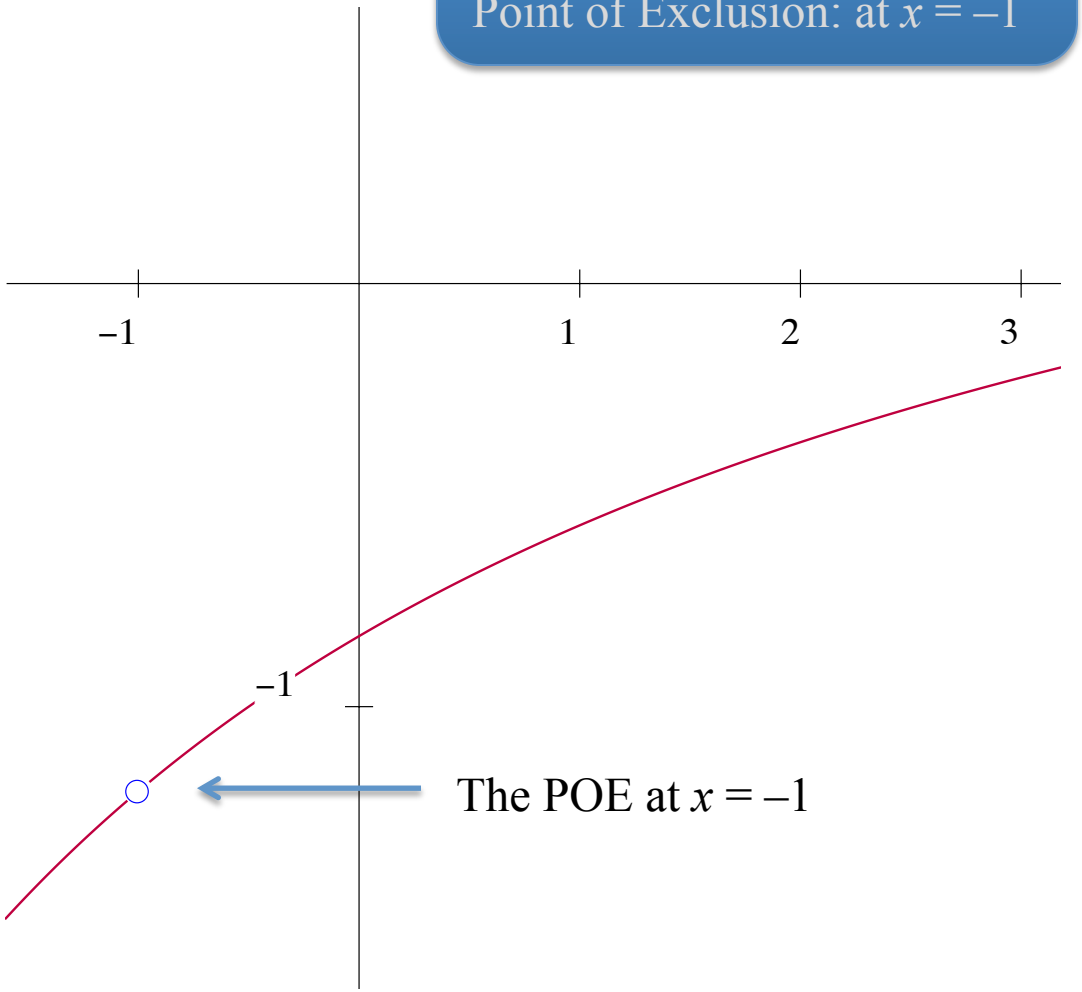
What's the difference between the two?

$$y = \frac{x^2 - 4x - 5}{x^2 + 7x + 6}$$

Zero: at $x = 5$

Vertical Asymptote: at $x = -6$

Point of Exclusion: at $x = -1$

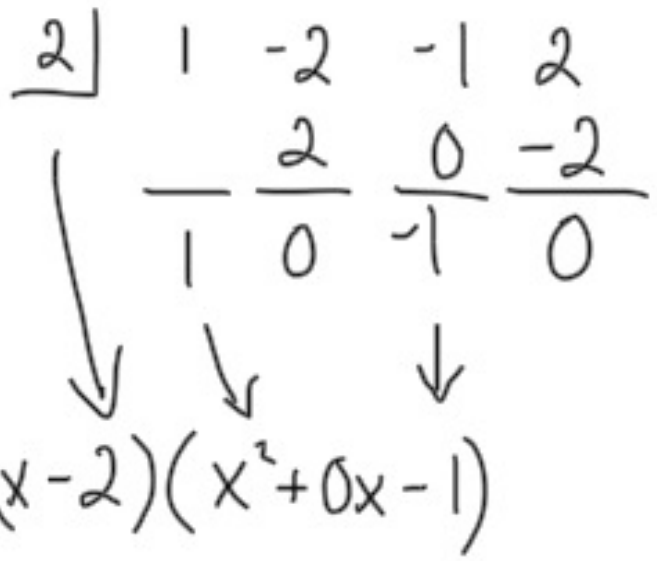


What's the difference between the two?

$$y = \frac{x^2 - 4}{x^3 - 2x^2 - x + 2}$$

Synthetic division (1h)

Or factoring by grouping



$$y = \frac{(x-2)(x+2)}{(x-2)(x^2-1)}$$

$$y = \frac{(x-2)(x+2)}{(x-2)(x-1)(x+1)}$$

Where the numerator = 0

$$(x-2)(x+2) = 0$$

$$x = \pm 2$$

Where the denominator = 0

$$(x-2)(x-1)(x+1) = 0$$

$$x = \pm 1, 2$$

Remember that ±2 are good roots to try because the last term is 2

The last term means that ±1 and ±2 are your best options

Since we're looking for roots to cancel with the numerator term, ±2 is a good place to start.

$$y = \frac{x^2 - 4}{x^3 - 2x^2 - x + 2}$$

$$y = \frac{(x-2)(x+2)}{(x-2)(x^2-1)}$$

$$y = \frac{(x-2)(x+2)}{(x-2)(x-1)(x+1)}$$

Where the numerator = 0

$$(x-2)(x+2) = 0$$

$$x = \pm 2$$

Where the denominator = 0

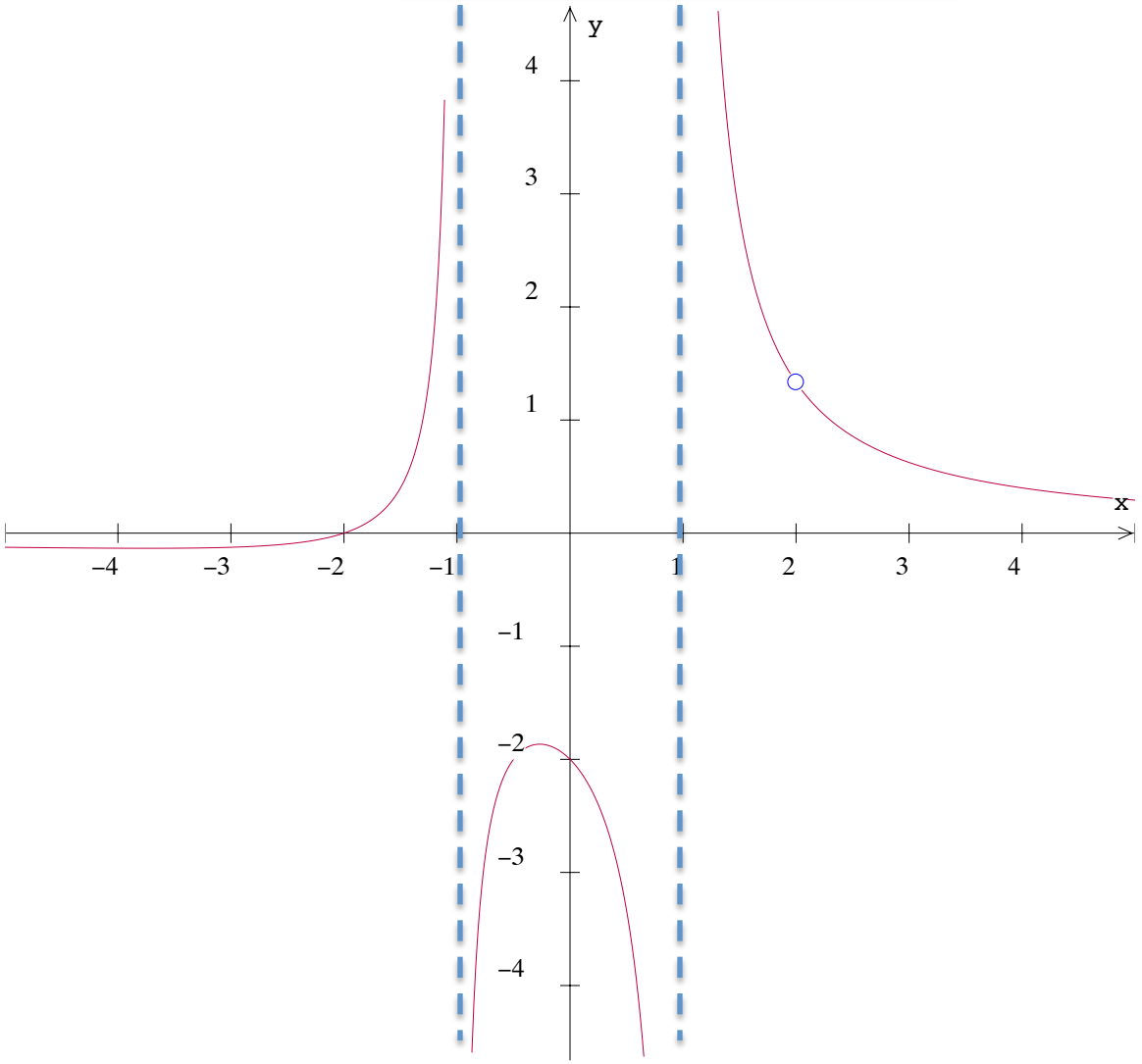
$$(x-2)(x-1)(x+1) = 0$$

$$x = \pm 1, 2$$

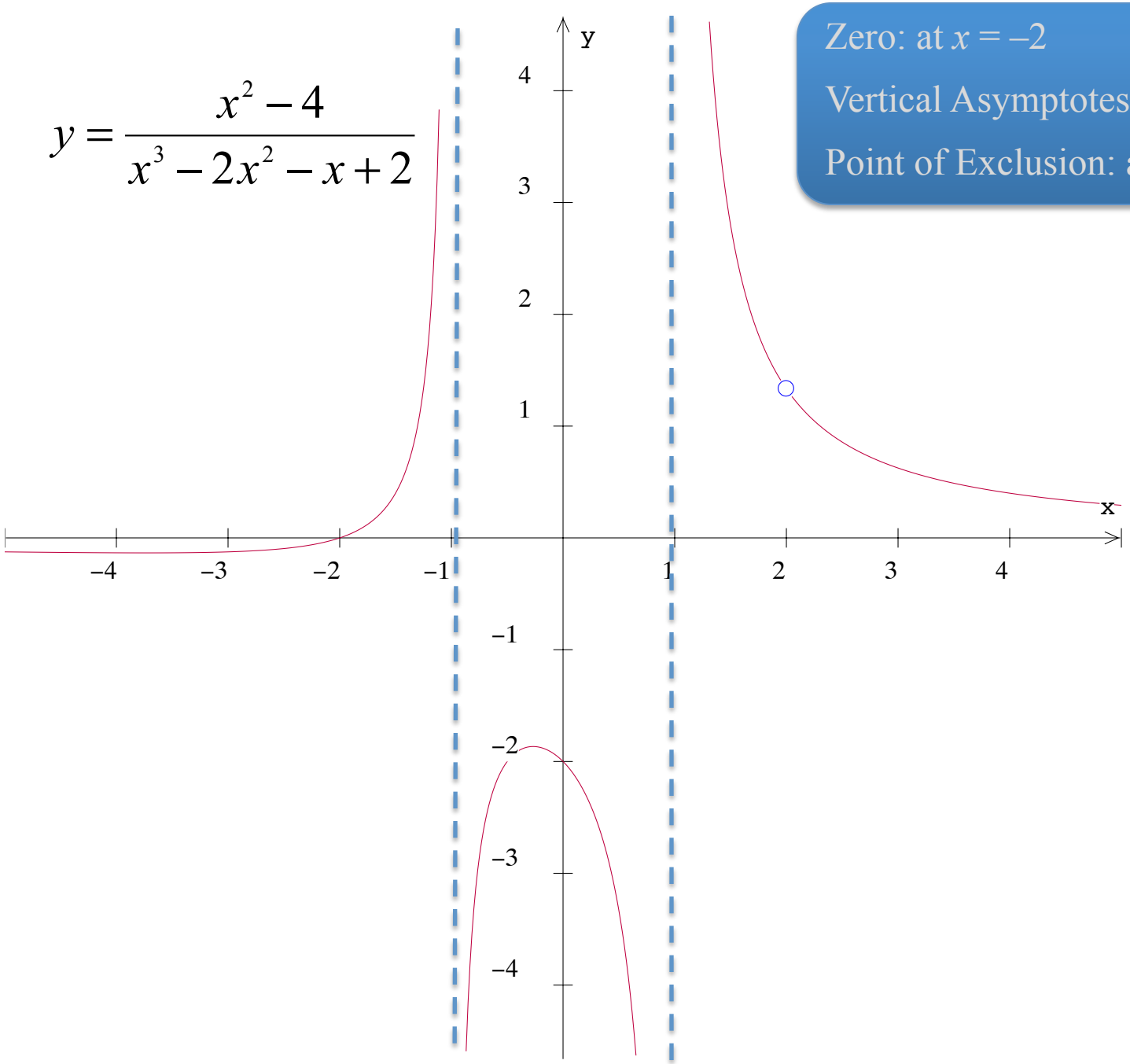
Zero: at $x = -2$

Vertical Asymptotes: at $x = \pm 1$

Point of Exclusion: at $x = 2$



$$y = \frac{x^2 - 4}{x^3 - 2x^2 - x + 2}$$



Zero: at $x = -2$

Vertical Asymptotes: at $x = \pm 1$

Point of Exclusion: at $x = 2$

The End