The Quotient Rule

Easy from top to bottom

Remember the Power Rule?   Of course you do
And not just because it's so easy to remember
but also...

...because it's 5d: Find the derivative using the
Power Rule

But now we want to build on this standard to
master two more:

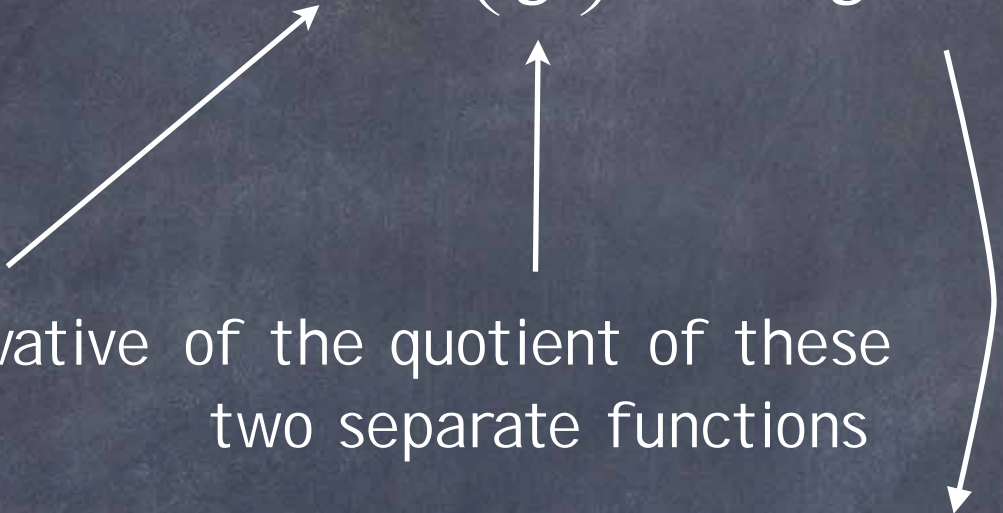Standard 7c: Find the derivative of a rational
function using the Quotient Rule

Standard 7d: Apply sign patterns to the first
derivative

The Quotient Rule

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - g'f}{g^2}$$

Huh?

Take the derivative of the quotient of these
two separate functions

Derivative of the top times the bottom minus
the derivative of the bottom times the top...

...all divided by the bottom squared

...or as some have said to help them remember...

"Low D-High, High D-Low, square the bottom and off you go"

Of course, nothing tells the story better than examples:

The Quotient Rule

$$f = x^2 + 5x - 1 \qquad\qquad\qquad\qquad\qquad g = 3x^2 - 4$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - g'f}{g^2}$$

$$f' = 2x + 5 \qquad\qquad\qquad\qquad\qquad\qquad g' = 6x$$

$$\frac{d}{dx}\left(\overbrace{\underbrace{\frac{x^2 + 5x - 1}{3x^2 - 4}}_{g}}^{f}\right) = \frac{\overbrace{(2x + 5)}^{f'}\overbrace{(3x^2 - 4)}^{g} - \overbrace{(6x)}^{g'}\overbrace{(x^2 + 5x - 1)}^{f}}{\underbrace{\left(3x^2 - 4\right)^2}_{g^2}} =$$

$$= \frac{6x^3 + 15x^2 - 8x - 20 - 6x^3 - 30x^2 + 6x}{\left(3x^2 - 4\right)^2} = \frac{-15x^2 - 2x - 20}{\left(3x^2 - 4\right)^2}$$

Let's try another one...

The Quotient Rule

$f = x^2 + 4$ $\hspace{6cm}$ $g = x$

$$\frac{d}{dx}\left(\frac{x^2 + 4}{x}\right) =$$

$f' = 2x$ $\hspace{6cm}$ $g' = 1$

$$\frac{d}{dx}\left(\frac{x^2 + 4}{x}\right) = \frac{\overbrace{(2x)}^{f'}\overbrace{(x)}^{g} - \overbrace{(1)}^{g'}\overbrace{(x^2 + 4)}^{f}}{\underbrace{x^2}_{g^2}} = \frac{2x^2 - (x^2 + 4)}{x^2} =$$

$$\frac{x^2 - 4}{x^2}$$

Its that simple

The Quotient Rule

So now that we know that

$$\frac{d}{dx}\left(\frac{x^2+4}{x}\right) = \frac{x^2-4}{x^2}$$

Let's find the critical points for $y = \left(\frac{x^2+4}{x}\right)$

We already know it's derivative so…

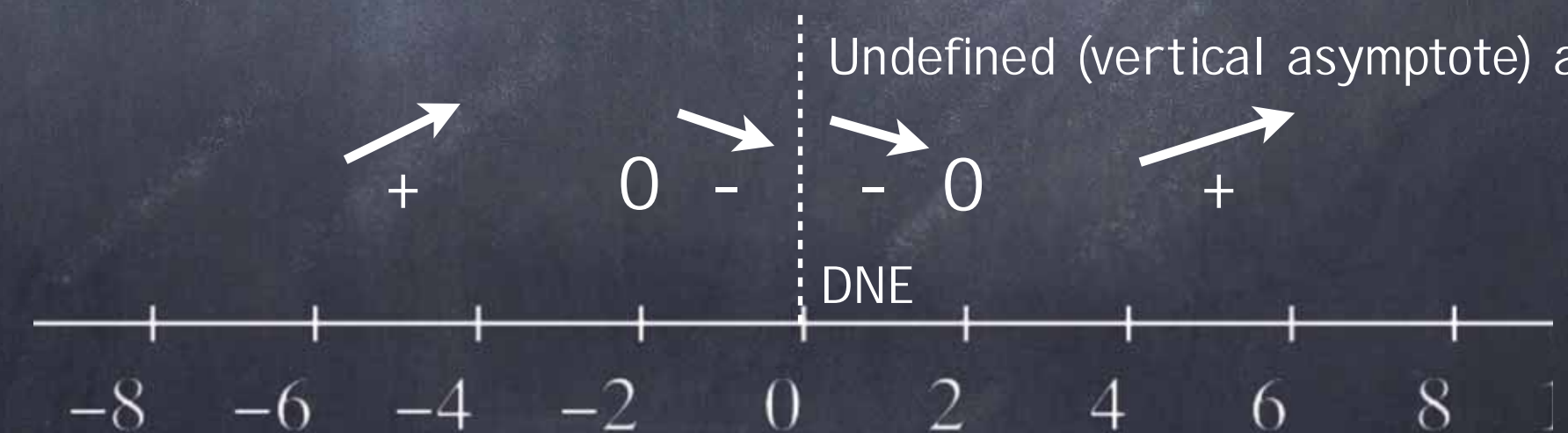$$\frac{x^2-4}{x^2} = 0 \longrightarrow x^2-4 = 0 \longrightarrow (x-2)(x+2) = 0$$

Undefined (vertical asymptote) at 0

+   0 -   - 0   +

$x = \pm 2$

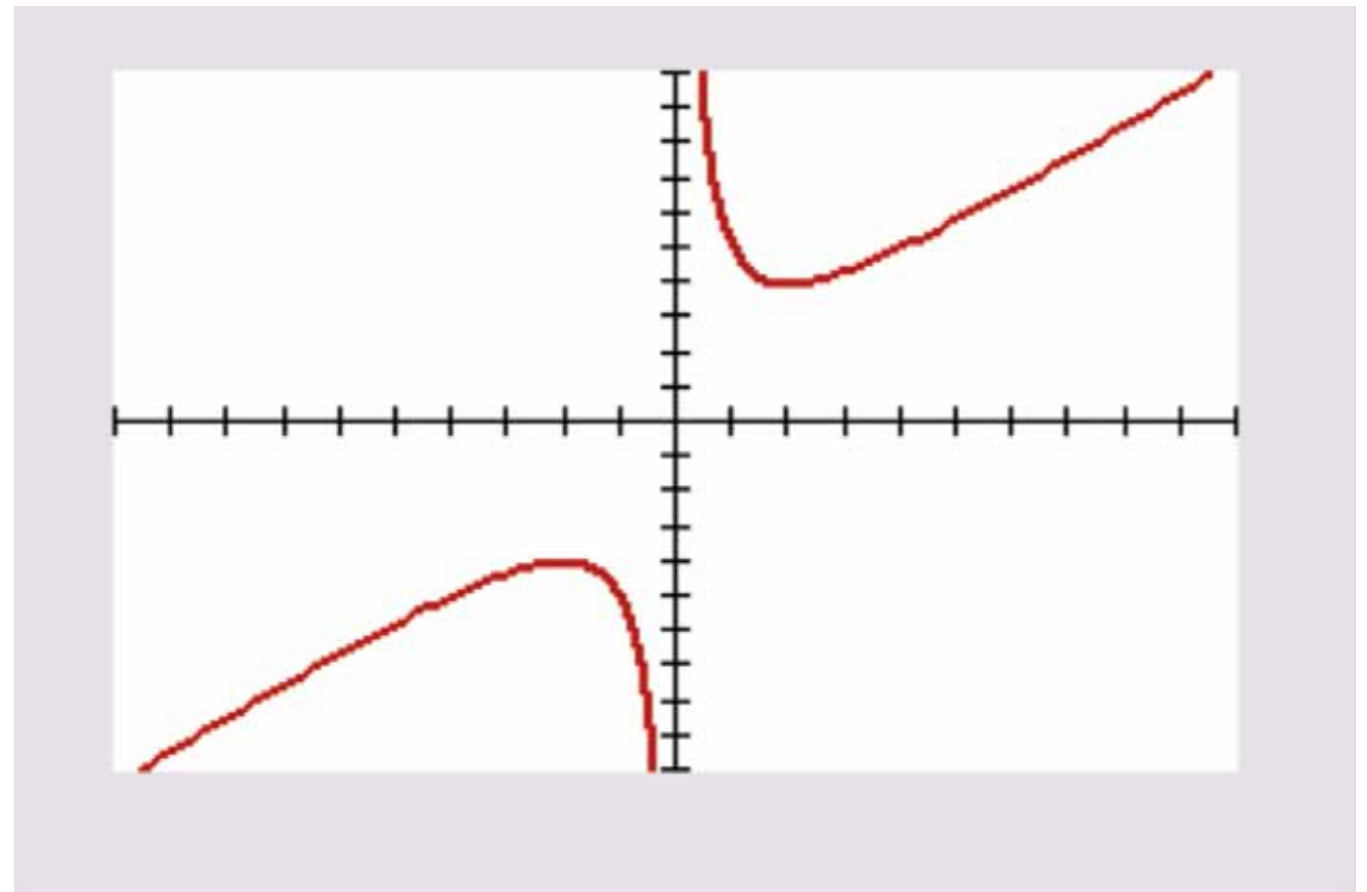DNE

Max at x = -2

−8   −6   −4   −2   0   2   4   6   8

Min at x = 2

It really is that simple

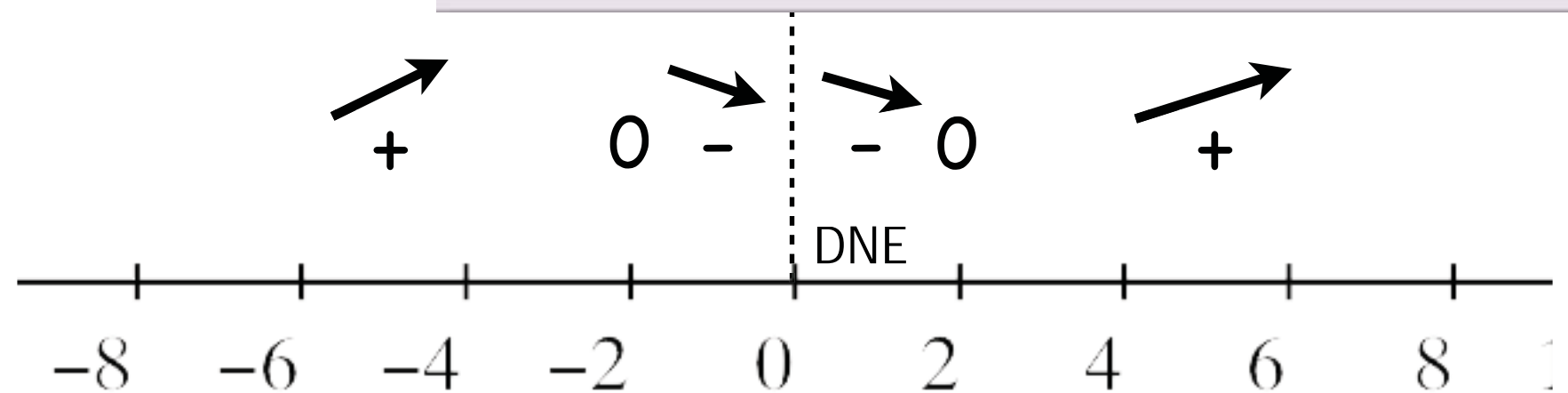It's important to remember to include the points at which the function is undefined when making your sign patterns

$$\frac{d}{dx}\left(\frac{x^2 + 4}{x}\right) = \frac{x^2 - 4}{x^2}$$

$x = \pm 2$

Max at x = -2

Min at x = 2

It really is that simple